Starter

1) Fully factorise $4y^2 - 5y - 6$

$-8y + 3y$ $(4y+3)(y-2)$

2) Expand the brackets and simplify: $(m + 4)(2m - 3)$

$2m^2 - 3m + 8m - 12$

$2m^2 + 5m - 12$

3) Calculate 20% of 340 without a calculator.

$34 \times 2 = 68$

4) What is 40ml increased by 20% ?

48ml

Today's Learning:

To find the equation of quadratic graphs using substitution of a point.

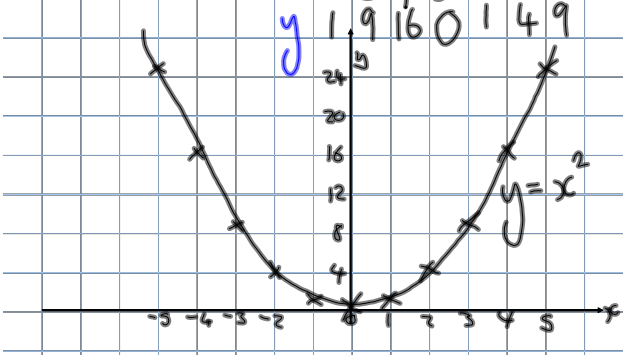
Quadratic Graphs

A quadratic equation involves a squared term

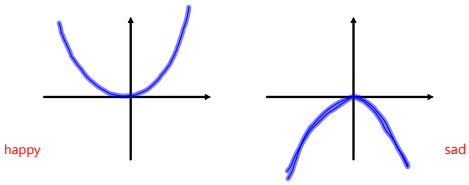
e.g. $3x^2 + 2x - 3 = 0$

The simplest quadratic graph is $y = x^2$

x	3	4	0	-1	-2	-3
y	9	16	0	1	4	9



The graph of $y = kx^2$



Positive k Negative k

$y = kx^2$ graph is stretched ↕ by a factor of k

Starter

1) Factorise fully: $2x^2 + 1x - 10$ $(2x+5)(x-2)$

$-5 \quad +4$

$5 \quad -4$

2) Without a calculator, find $2.3 \times 10^5 \times 3 \times 10^{-2}$

$= 2.3 \times 3 \times 10^5 \times 10^{-2}$

$= 2.3 \times 3 \times 10^3$

$= 6.9 \times 10^3$

3) Without a calculator, simplify $\frac{912}{18} = \frac{456}{9} = \frac{152}{3}$

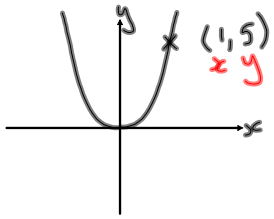
$9\overline{)456}$ r 050

$3\overline{)456}$ 152

Today's Learning:

To continue to consider transformations of quadratic graphs.

e.g. Find the equation of the graph of the form $y = kx^2$



$5 = k \times 1^2$

$5 = k \times 1$

$k = 5$

$y = 5x^2$

Starter

1) Without a calculator, find a fifth of 70. 14

2) Fully factorise: $3g^2 - 13g - 10$ $(3g+2)(g-5)$

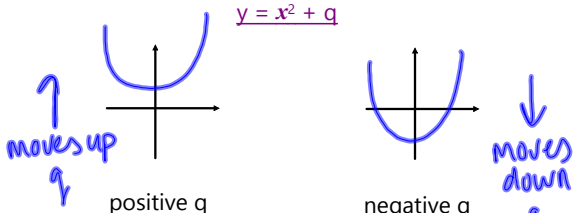
3) Multiply out the brackets: $(e + 2)(e + 3)(e - 1)$

$3g^2 - 10g - 3g + 10$

$(3g+2)(g-5)$

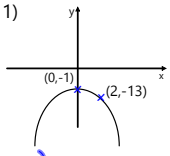
$3g^2 + 2g - 15g - 10$

$y = x^2 + q$



e.g. Find k and q from the graphs of $y = kx^2 + q$:

1)



$(0,-1)$

$y = kx^2 + q$

$-1 = k(0)^2 + q$

$-1 = q$

$y = kx^2 - 1$

$-13 = k \times 2^2 - 1$

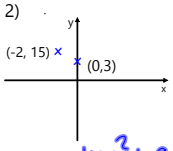
$-13 = 4k - 1$

$-12 = 4k$

$-3 = k$

$y = -3x^2 - 1$

2)



$y = kx^2 + q$

$3 = k \times 0^2 + q$

$3 = 0 + q$

$3 = q$

$y = kx^2 + 3$

$15 = k \times (-2)^2 + 3$

$15 = 4k + 3$

$12 = 4k$

$3 = k$

$y = 3x^2 + 3$

Starter

1) Fully factorise: $3m^2 + 12m + 9$ $3(m+1)(m+3)$

$3(m^2 + 4m + 3)$ $(3m+3)(m+3)$

$3m^2 + 9m + 3m + 9$

2) Simplify the following:

a) $\sqrt{40} + \sqrt{160}$

$= \sqrt{4 \times 10} + \sqrt{16 \times 10}$

$= 2\sqrt{10} + 4\sqrt{10} = 6\sqrt{10}$

b) $\frac{x^4}{x^2 \times x} = \frac{x^4}{x^3}$

$= x$

3) Without a calculator, find 53 x 31

$53 \times 3 = 159$

1590

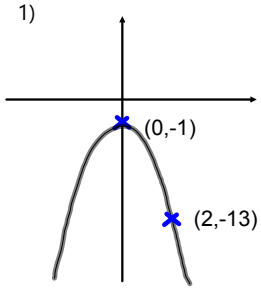
$+53$

1643

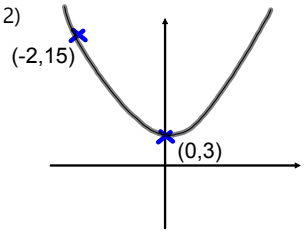
$53 \times 31 = 1643$

Find the equations of these graphs, of the form $y = kx^2 + q$

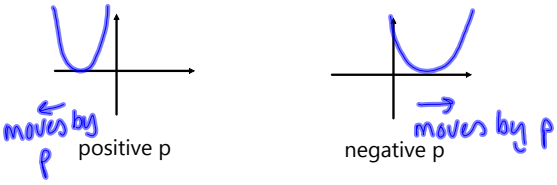
1)



2)

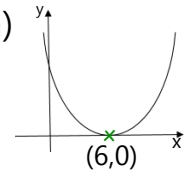


The graph of $y = (x + p)^2$



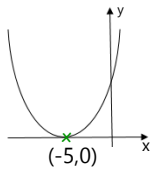
e.g. Find p for these graphs of $y = (x + p)^2$:

a)



$p = -6$

b)



$p = 5$

Starter

1) Find a and b, given:

$2a - b = 2$

$a + b = 7$

$3a = 9$

$a = 3$

$b = 4$

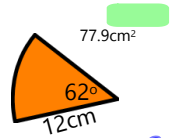
2) Calculate $3 \times 10^4 \times 7 \times 10^2$, giving your answer in scientific notation

21×10^6

$= 2.1 \times 10^7$

3) Round to 3 sig. fig.

4) Find the area of the sector:



$\frac{x}{360} \times \pi \times r^2$

$\frac{62}{360} \times \pi \times 12^2$

Sketching Quadratic Graphs

We can be asked to label:

- Turning Point and its nature
- Roots (where it crosses the x-axis)
- y-intercept
- Equation of the axis of symmetry

e.g. 1) Sketch $y = -(x + 3)^2$ and label all of the above.

TP: (-3,0) maximum

Roots: (-3,0)

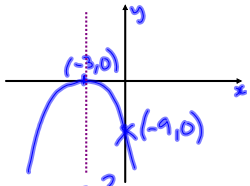
y-intercept: Set $x = 0$

$y = -(0+3)^2$

$= -(3)^2$

$= -9$

(-9,0)



$x = -3$

- eq^n of axis of sym

Starter $x^2 + x - 6x - 6$

1. Factorise: $x^2 - x - 6$ $(x-3)(x+2)$
2. Factorise: $x^2 - 25$ $(x+5)(x-5)$

$x^2 - 2x + 3x - 6$

3. Factorise: $2x^2 - 8x$ $2(x^2 - 4x) = 2x(x-4)$
4. State the gradient of the line: $4y + 12 = 2x$

e.g. 2) Sketch the graph of $y = -(x + 1)^2 - 5$

TP @ (-1,-5) - maximum

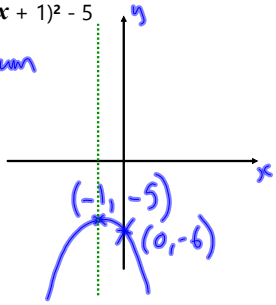
y-intercept: $x = 0$

$y = -(0+1)^2 - 5$

$= -1 - 5 = -6$

(0,-6)

No roots



$x = -1$

line of symmetry

Starter

1) Write down the y-intercept of the line $2y = 3 - 2x$

$2y = -2x + 3$

$y = -x + 1.5$

$2y = 3$

$y = 1.5$

2) Without a calculator, find a fifth of 22 4.4

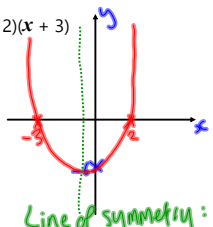
3) Simplify $3e^4 \times 2e^{-2}$ $6e^2$

4) What is the difference between -4 and 7? 11

$a \times b = 0$

What can you say about a and b?

e.g. 3) Sketch the graph of $y = (x - 2)(x + 3)$



y-int: set $x = 0$

$y = (0-2)(0+3)$

$= (-2)(3)$

$= -6$

Roots: set $y = 0$

$0 = (x-2)(x+3)$

$x - 2 = 0$ or $x + 3 = 0$

$x = 2$ or $x = -3$

2.5
1.5
0.5
0
-0.5

Line of symmetry:

TP $x = -0.5$

$y = (x-2)(x+3)$

$y = (-0.5-2)(-0.5+3)$

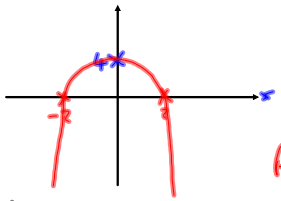
$= (-2.5)(2.5)$

$= -6.25$

TP @ $(-0.5, -6.25)$

minimum

Sketch the graph of $y = -(x + 2)(x - 2)$



Line of symmetry

$x = 0$

TP @ $(0, 4)$

maximum

y-int: set $x = 0$

$y = -(2)(-2)$

$= -(-4)$

$= 4$

Roots: set $y = 0$

$0 = -(x+2)(x-2)$

$x + 2 = 0$ or $x - 2 = 0$

$x = -2$ or $x = 2$

Starter

Factorise the following:

1) $3m^2 - 13m - 10$ 2) $2p^2 - 18$ c) $3gh + 6g^2$

$(3m+2)(m-5)$ $2(p^2-9)$ $3g(h+2g)$

$3m^2 + 2m - 15m - 10$

$2(p+3)(p-3)$

Today's Learning:

Sketching quadratic graphs.

Sketch y = (x + 4)(x - 8)

y-int: set x = 0

y = (0+4)(0-8)

= (4)(-8)

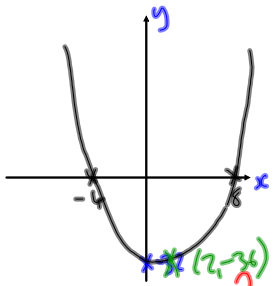
= -32

Roots: set y = 0

0 = (x+4)(x-8)

x+4 = 0 or x-8 = 0

x = -4 or x = 8



line of sym: x = 2

TP: y = (2+4)(2-8)

= (6)(-6)

= -36

(2, -36)

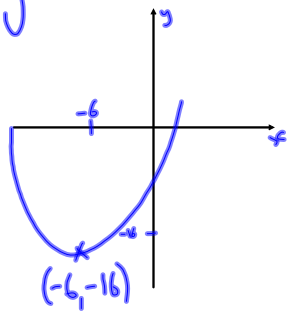
minimum

Starter

a) Write the expression (x + 10)(x + 2) in completed square form. $= x^2 + 12x + 20 = (x+6)^2 - 16$

b) Hence sketch the graph y = (x + 10)(x + 2), marking the coordinates of the turning point and the nature of the turning point.

$y = (x+6)^2 - 16$



$y = (x - 1)^2 - 25$ Spot the mistake(s)!

Roots: $0 = x^2 - 2x - 24$

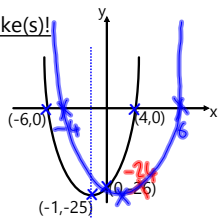
$= (x + 4)(x - 6)$

x = 4 or -6

x = -4 or 6

y intercept: $y = (-1)^2 - 25$ = 1-25

$= -26$ -24



Equation of axis of symmetry: x = -1 x = 1

TP occurs at (-1, -25) and is a minimum because $x^2 > 0$

(1, -25)

a × b = 0

How do we solve (x + 4)(x - 1) = 0 for x?

x + 4 = 0 or x - 1 = 0

x = -4 or x = 1

How might we solve $x^2 - x - 6 = 0$

6

1, 6

3, 2

(x - 3)(x + 2) = 0

x = 3 or -2

Solving Quadratic Equations 4/9/17

A quadratic equation can be written as $ax^2 + bx + c = 0$

Then, we can solve by factorising.

Examples:

35

7, 5

1, 35

1) $x^2 - 2x - 35 = 0$

(x - 7)(x + 5) = 0

x = 7 or x = -5

2) $2x^2 + 10x = 0$

2x(x + 5) = 0

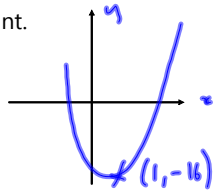
2x = 0 or x + 5 = 0

x = 0 x = -5

Starter $x^2 - 2x + 1$

a) Write the expression (x - 5)(x + 3) in completed square form. $x^2 - 2x - 15 = (x-1)^2 - 16$

b) Hence sketch the graph y = (x - 5)(x + 3), marking the coordinates of the turning point and the nature of the turning point.



$y = (x-1)^2 - 16$

minimum TP

Example:

Solve $2x^2 + 5x + 3 = 0$

$(2x+1)(x+3)=0$

$2x^2+6x+1x+3$ 3 1/3

$(2x+3)(x+1)=0$

$2x^2+2x+3x+3$

$2x+3=0$ or $x+1=0$

$-3\ \ -3$ $x=-1$

$2x=-3$

$x=-1.5$

Today's Learning:

To write any quadratic equation in the form **$ax^2 + bx + c = 0$** and to solve equations that don't factorise by using the **quadratic formula**.

The Quadratic Formula

If we have an equation $ax^2 + bx + c = 0$ that we can't factorise, we can use the Quadratic Formula to find solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ (given in exams)

Examples:

1) $x^2 - 5x - 14 = 0$

$a=1\ b=-5\ c=-14$

$x=\frac{5\pm\sqrt{(-5)^2-4(1)(-14)}}{2\times 1}$

$-4\times1\times-14$

-4×-14

$=56$

$x=\frac{5\pm\sqrt{25+56}}{2}$

$=\frac{5\pm\sqrt{81}}{2}$

$=\frac{5\pm9}{2}$

$x=\frac{5+9}{2}=\frac{14}{2}=7$

or $x=\frac{5-9}{2}=\frac{-4}{2}=-2$

2) $x^2+4x+1=0$ $c=1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-4\pm\sqrt{4^2-4(1)(1)}}{2(1)}$

$=\frac{-4\pm\sqrt{16-4}}{2}$

$=\frac{-4\pm\sqrt{12}}{2}$

$x=-0.27$ or -3.73 (2dp)

Starter

Solve using the Quadratic Formula, giving answers to 2 decimal places:

a) $4x^2 - 11 = 0$

$=\frac{\pm\sqrt{11}}{2}$

b) $(x+5)^2 = 7$ $(x+5)(x+5)$

$x^2+10x+25=7$

$x^2+10x+18=0$

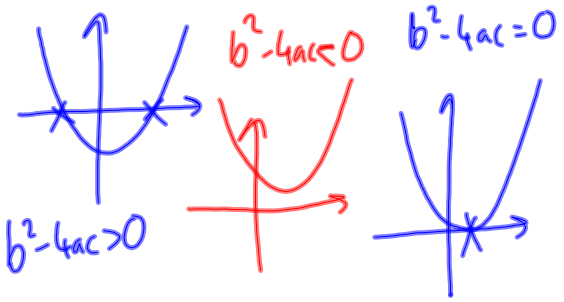
$x=-2.35$ or -7.65

c) $4x(x-3)+2=0$

$4x^2-12x+2=0$

$=2.82$ or 0.18

How can we tell how many roots an equation has?



The Discriminant

For a quadratic equation $ax^2 + bx + c = 0$ the discriminant is $b^2 - 4ac$.

$b^2 - 4ac > 0$ means 2 real, distinct roots

$b^2 - 4ac = 0$ means 2 real, equal roots

$b^2 - 4ac < 0$ means no real roots

e.g. 1) Determine the nature of the roots of $2(x + 1) = x^2 - 3$

$2x+2=x^2-3$

$-2x\ \ -2x$

$2=x^2-2x-3$

$-2\ \ -2$

$0=x^2-2x-5$

$a=1\ b=-2\ c=-5$

$b^2-4ac=4-4(1)(-5)$

$=4+20=24$

$b^2-4ac=24>0 \rightarrow$ 2 real, distinct roots.

e.g. 2) Find the range of values for T such that $x^2 + 2x - 2T = 0$ has 2 real, distinct roots.

$a = 1 \quad b = 2 \quad c = -2T$

$b^2 - 4ac = 4 - 4(1)(-2T)$

$= 4 + 8T > 0$

$-4 \qquad -4$

$8T > -4$

$\div 8 \qquad \div 8$

$T > \frac{-4}{8}$

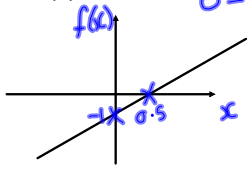
$T > \frac{-1}{2}$

Starter

1) Given $f(x) = x^2 - 4$, evaluate f(3)

2) Sketch the graph $f(x) = 2x - 1$. Write the coordinates where this line meets the line $f(x) = 4$.

$0 = 2x - 1$



3) Given $f(x) = 3x + 10$, find x such that $f(x) = 14.8$.

Starter

Rationalise the denominator:

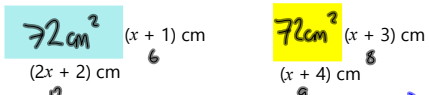
$$\frac{4}{(\sqrt{5} + \sqrt{2})} \times \frac{(\sqrt{5} - \sqrt{2})}{(\sqrt{5} - \sqrt{2})}$$

$$= \frac{4\sqrt{5} - 4\sqrt{2}}{5 - \sqrt{10} + \sqrt{10} - 2}$$

$$= \frac{4\sqrt{5} - 4\sqrt{2}}{3}$$

The **areas** of these rectangles are equal.

a) Find the value of x. b) Calculate the area of the rectangles.



$(2x+2)(x+1) = (x+3)(x+4)$

$2x^2 + 2x + 2x + 2 = x^2 + 4x + 3x + 12$

$2x^2 + 4x + 2 = x^2 + 7x + 12$

$-x^2 - 7x - 12 \qquad -x^2 - 7x - 12$

$x^2 - 3x - 10 = 0$

$(x - 5)(x + 2) = 0$

$x - 5 = 0 \text{ or } x + 2 = 0$

$x = 5 \text{ or } -2$

$x = -2$ not possible

so $x = 5$

Starter

1) Given the function $f(x) = (5 - x)^2$, evaluate:

a) f(3) b) f(-1)

a) $(5-3)^2$

$= 2^2 = 4$

b) $f(-1) = (5 - -1)^2$

$= 6^2 = 36$

2) Multiply out the brackets and simplify:

(w + 1)(w - 1)(w + 5)

$(w^2 - 1)(w + 5)$

$= w^3 - w + 5w^2 - 5$

$= w^3 + 5w^2 - w - 5$

Starter

Simplify:

$$\frac{(a^2)^3 \times a^{-2}}{a^5 \times a^{-5}}$$

$$= \frac{a^6 \times a^{-2}}{a^5 \times a^{-5}} = \frac{a^4}{a^0}$$

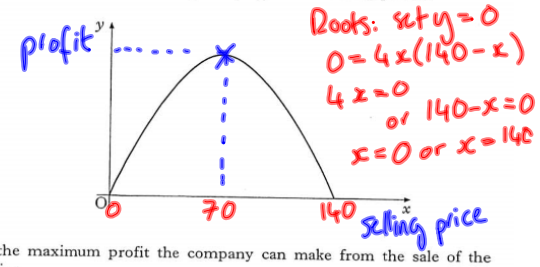
$$= \frac{a^4}{1} = a^4$$

The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x),$$

where y is the profit (in pounds) and x is the selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x.



Roots: set $y = 0$

$0 = 4x(140 - x)$

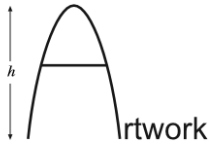
$4x = 0$ or $140 - x = 0$

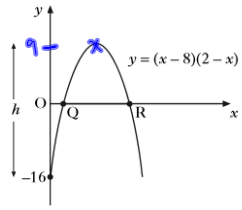
$x = 0$ or $x = 140$

Find the maximum profit the company can make from the sale of the magazine.

The curved part of the letter A in the *Artwork* logo is in the shape of a parabola.

The equation of this parabola is $y = (x - 8)(2 - x)$.





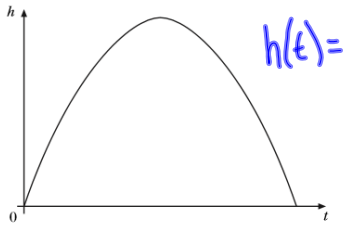
(*a*) Write down the coordinates of Q and R.

(*b*) Calculate the height, h, of the letter A. 25

The diagram below shows the path of a rocket which is fired into the air.

The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$



$h(t) = -2t(t - 14)$

(*a*) For how many seconds is the rocket in flight?

(*b*) What is the maximum height reached by the rocket?