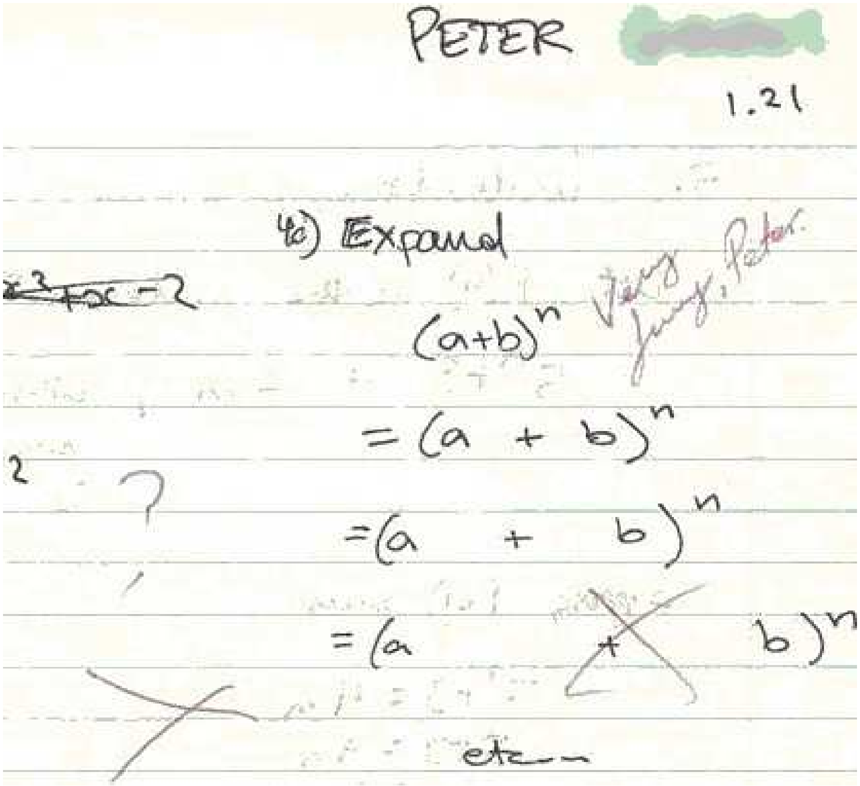
S4 National 5 Maths

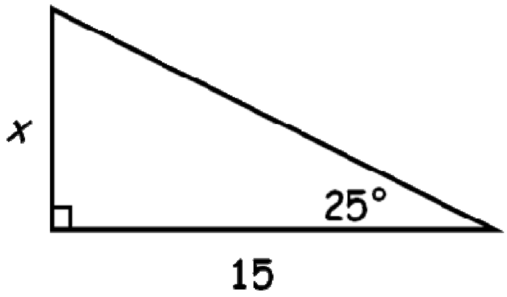


Trigonometry & Triangle Calculations

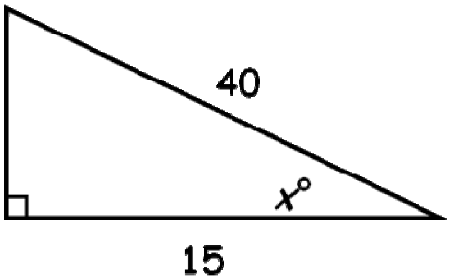
Revision

In 3rd year we did the first part of trigonometry which was SOHCAHTOA. There were three types of equations which we had to solve using trigonometry. The first two were in the credit and general but the last was only in credit.

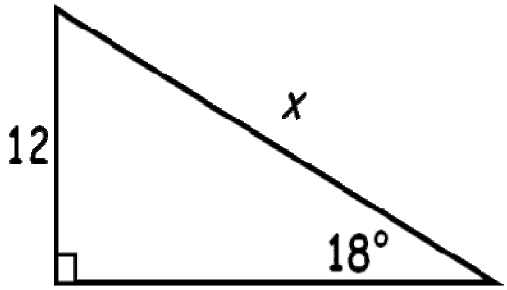
Ex1 Find x.



Ex2 Find x.



Ex3 Find x.

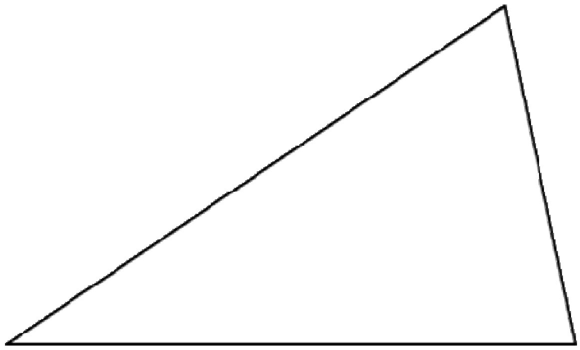


The Sine Rule

If we have a right angled triangle we have seen that we can use SOHCAHTOA or Pythagoras to find any missing sides or angles.

If we don't have a right angled triangle the only method we have had to find missing sides and angles is to draw a scale drawing.

In the credit exam no marks can be given for answers obtained from a scale drawing so we must use another method.

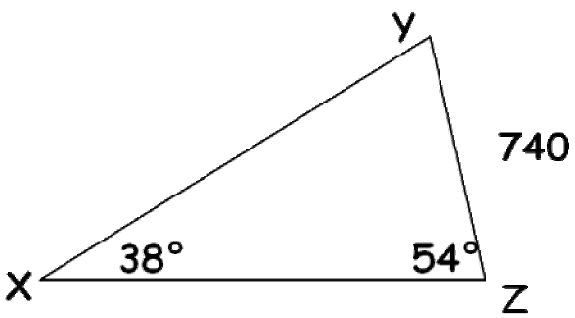


- The vertices of a right angled triangle are named using upper case letters.
- The sides opposite these vertices are named using the same letters but lower case.

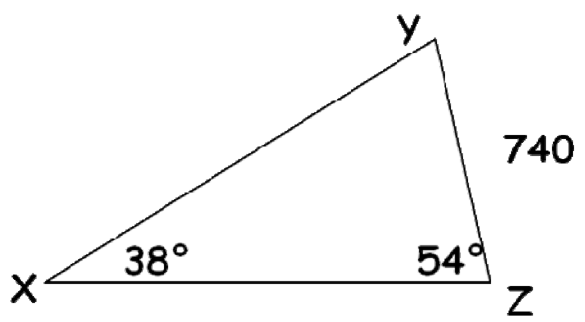
The sine rule is then:

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

Ex1



Find the length of the side XY.



$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$ so we use:

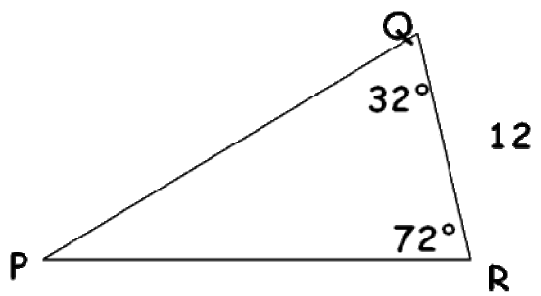
$$\frac{x}{\sin X} = \frac{z}{\sin Z}$$

$$\frac{740}{\sin 38} = \frac{z}{\sin 54}$$

$$z = \frac{740 \times \sin 54}{\sin 38}$$

$$\underline{\underline{z = 972}}$$

Ex2



Find the length of the side PR.

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R}$$ so we use:

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{12}{\sin 76} = \frac{q}{\sin 32}$$

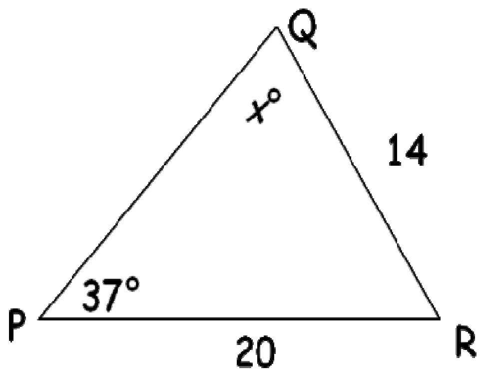
$$q = \frac{12 \times \sin 32}{\sin 76}$$

$$\underline{\underline{q = 6.6}}$$

Calculating an Angle Using the Sine Rule

To calculate an angle using the sine rule we still use the same formula.

Ex1



Find x.

$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} \quad \text{so we use:} \quad \frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{p}{\sin P} = \frac{q}{\sin Q}$$

$$\frac{14}{\sin 37} = \frac{20}{\sin x}$$

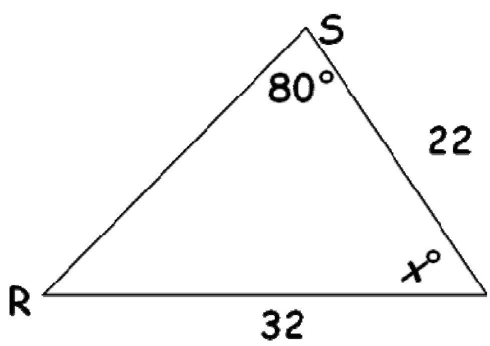
$$14 \times \sin x = 20 \times \sin 37$$

$$\sin x = \frac{20 \times \sin 37}{14}$$

$$\sin x = 0.860$$

$$\underline{\underline{x = 59.3}}$$

Ex2



Find x.

$$\frac{r}{\sin R} = \frac{s}{\sin S} = \frac{t}{\sin T}$$

It appears that we cannot use the sine rule as we do not have two pairs.

We could work out angle R and use it to find the missing angle T.

To find angle R:

$$\frac{r}{\sin R} = \frac{s}{\sin S}$$

$$\frac{22}{\sin R} = \frac{32}{\sin 80}$$

$$22 \times \sin 80 = 32 \times \sin R$$

$$\frac{22 \times \sin 80}{32} = \sin R$$

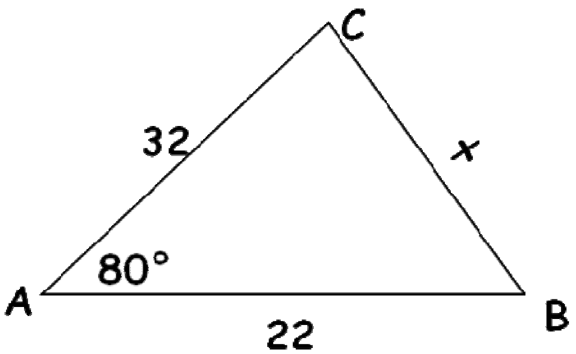
$$0.677 = \sin R$$
$$\underline{\underline{42.6 = R}}$$

So to find x (Angle T):

$$x = 180 - 80 - 42.6$$
$$\underline{\underline{x = 58.4^\circ}}$$

<u>The Cosine Rule</u>

Consider the following:



Sine Rule gives us:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

From this we can see that we can't use the sine rule
When this happens we use the **cosine rule.**

The Cosine Rule is:

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$$

The cosine rule can be written using any of the three sides.

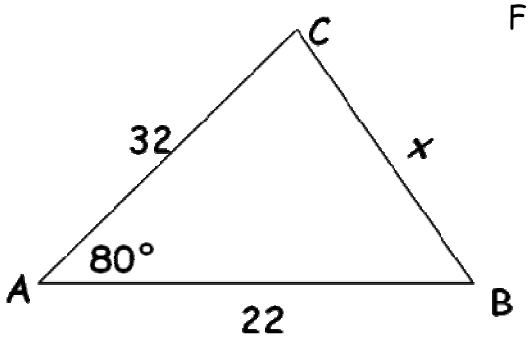
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Notice also that if we have a right angle at A the cosine rule becomes:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = b^2 + c^2 - 2bc \times \cos 90$$
$$a^2 = b^2 + c^2 - 2bc \times 0$$
$$\underline{\underline{a^2 = b^2 + c^2}}$$

This is Pythagoras Theorem!!!!

Ex1



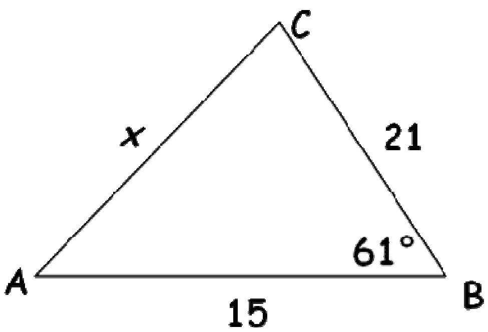
Find x.

Using Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = 32^2 + 22^2 - 2 \times 32 \times 22 \times \cos 80$$
$$x^2 = 1024 + 484 - 244.5$$
$$x^2 = 1263.5$$
$$x = \sqrt{1263.5}$$
$$\underline{\underline{x = 35.5}}$$

Ex2

Find x.

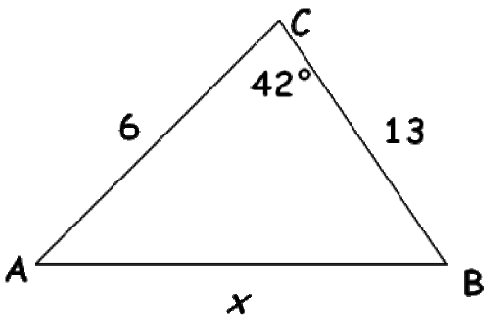


Using Cosine Rule:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$x^2 = 21^2 + 15^2 - 2 \times 21 \times 15 \times \cos 61$$
$$x^2 = 441 + 225 - 305.43$$
$$x^2 = 360.57$$
$$x = \sqrt{360.57}$$
$$\underline{\underline{x = 18.99}}$$

Ex3

Find x.



Using Cosine Rule:

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$x^2 = 13^2 + 6^2 - 2\times 13\times 6\times \cos 42$$
$$x^2 = 169 + 36 - 115.93$$
$$x^2 = 89.1$$
$$x = \sqrt{89.1}$$
$$\underline{\underline{x = 9.4}}$$

Calculating Angles Using the Cosine Rule

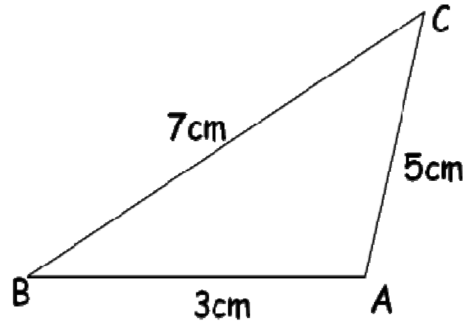
We can use the cosine rule to find an angle inside a triangle if we know the lengths of all three sides.

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{(The Cosine Rule)}$$
$$2bc\cos A = b^2 + c^2 - a^2$$
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Therefore the cosine rule becomes:

$$\boxed{\cos A = \frac{b^2 + c^2 - a^2}{2bc}}$$

Ex1
Calculate the size of the largest angle in triangle *ABC*.



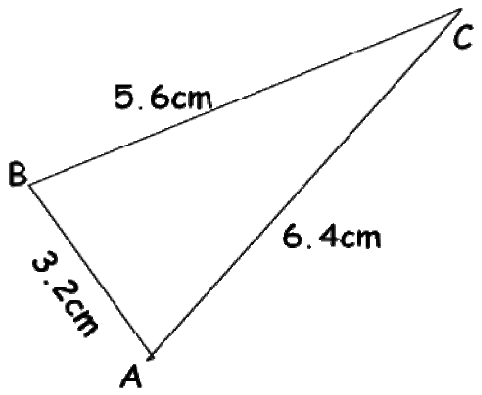
The largest angle is the angle opposite the longest side.

So Angle A is the one we want.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{5^2 + 3^2 - 7^2}{2 \times 5 \times 3} = \frac{-15}{30} = -0.5$$

$$\cos A = -0.5$$
$$A = \cos^{-1}(-0.5)$$
$$\underline{\underline{A = 120^{\circ}}}$$

Ex2
Calculate the size of the smallest angle in triangle *ABC*.



The smallest angle is the angle opposite the smallest side.

So Angle C is the one we want.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{5.6^2 + 6.4^2 - 3.2^2}{2 \times 5.6 \times 6.4} = \frac{62.08}{71.68} = 0.866$$

$$\cos A = 0.866$$
$$A = \cos^{-1}(0.866)$$
$$\underline{\underline{A = 29.99^{\circ} \text{ or } 30^{\circ}}}$$

The Area of a Triangle

From S1 we have all used the the formula for calculating the area of a triangle:

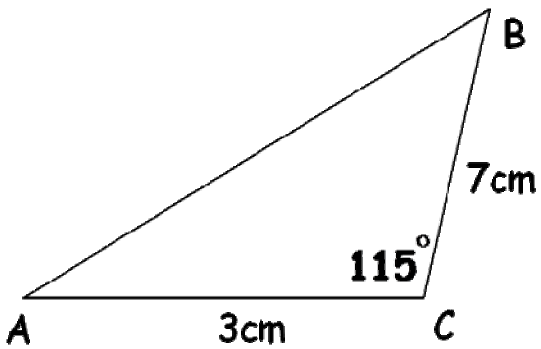
$$A = \frac{1}{2}bh$$

However in credit maths we have a new formula to calculate the area of a triangle when we don't know the height.

$$\boxed{A = \frac{1}{2}ab\sin C}$$

Ex1

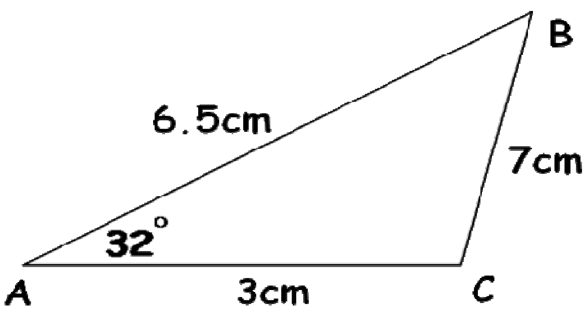
Calculate the area of triangle *ABC*.



$$Area = \frac{1}{2}ab\sin C = \frac{1}{2} \times 7 \times 3 \times \sin 115 = \underline{\underline{9.4cm^2}}$$

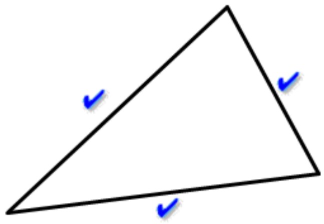
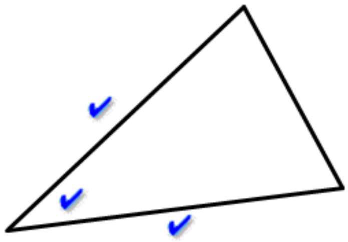
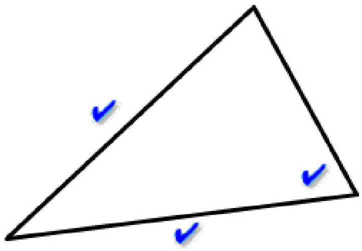
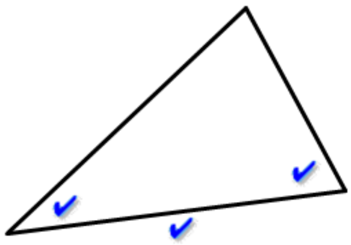
Ex2

Calculate the area of triangle *ABC*.



$$Area = \frac{1}{2}bc\sin A = \frac{1}{2} \times 3 \times 6.5 \times \sin 32 = \underline{\underline{5.2cm^2}}$$

Which Formula Do I Use?!?!?!

Given	**Sketch**	**Use**
Three sides		$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
Two sides and the angle between them		$a^2 = b^2 + c^2 - 2bc\cos A$
Two sides and the angle *not* between them		$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
One side and two angles		**Find the third angle then:** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$