Complex Numbers Exam Practice

1. Verify that i is a solution of $z^{4}+4z^{3}+3z^{2}+4z+2=0$.

Hence find all the solutions. (5)

1. Identify the locus in the complex plane given by $\left|z+i\right|$= 2.
2. Given that $w=cosθ+i sinθ$, show that $\frac{1}{w}=cosθ-i sinθ.$ (1)

Use de Moivre’s theorem to prove $w^{k}+ w^{-k}=2coskθ$, where k is a

Natural number. (3)

Expand $\left(w+ w^{-1}\right)^{4}$ by the binomial theorem and hence show that

 $cos^{4}θ= \frac{1}{8}\cos(4θ+ \frac{1}{2}\cos(2θ+\frac{3}{8}))$. (5)

1. Given the equation $ z+2iz=8+7i$, express *z* in the form *a + ib*. (4)
2. Let $z=cosθ+i sinθ$.
3. Use the binomial expansion to express $z^{4}$ in the form $u+iv$, where *u* and *v*

Are expressions involving *sin* $ϑ$ and *cos*$θ$*.* (3)

1. Use de Moivre’s theorem to write down an expression for $z^{4}$. (1)
2. Using the results of (a) and (b), show that

 $\frac{\cos(4θ)}{cos^{2}θ}=pcos^{2}θ+q sec^{2}θ+r$, where $-\frac{π}{2}< θ< \frac{π}{2}$ ,

 Stating the values of p, q and r. (6)

1. Express the complex number $z= -i+\frac{1}{1-i}$, in the form $z=x+iy$, stating

the values of x and y. (3)

Find the modulus and argument of z and plot z and z on an Argand diagram (4)



7.

8. Given that $\left|z-2\right|=\left|z+i\right|$ where $z=x+iy$, show that $ax+by+c=0$ for suitable

 values of a, b and c. (3)

Indicate on an Argand diagram the locus of complex numbers z which satisfy $\left|z-2\right|=\left|z+i\right|$.

 ( 1)