Functions/Graphs Past Papers Unit 1 Outcome 2

Written Questions

[SQA] 1. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x}, x \neq 0$.

(a) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(b) If $q(x) = \dfrac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(b)	2	C	CN	A4	x	
(b)	1	A/B	CN	A4		

- $\bullet^1$ ic: interpret composite func.
- $\bullet^2$ pd: process

- $\bullet^3$ ic: interpret composite func.
- $\bullet^4$ pd: process
- $\bullet^5$ pd: process

- $\bullet^1$ $f\left(\frac{3}{x}\right)$ *stated or implied by* $\bullet^2$
- $\bullet^2$ $3 - \frac{3}{x}$
- $\bullet^3$ $p\left(\frac{3}{3-x}\right)$ *stated or implied by* $\bullet^4$
- $\bullet^4$ $3 - \dfrac{3}{\frac{3}{3-x}}$
- $\bullet^5$ x

[SQA] 2. The diagram illustrates three functions f, g and h. The functions are defined by $f(x) = 2x + 5$ and $g(x) = x^2 - 3$.

The function h is such that whenever **2**
$f(p) = q$ and $g(q) = r$ then $h(p) = r$. **2**
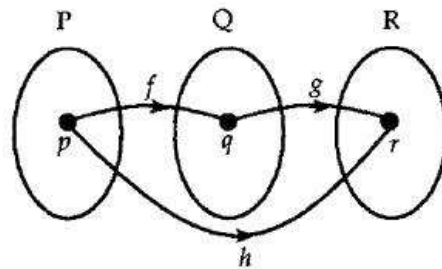
O (a) If $q = 7$, find the values of p and r.

x (b) Find a formula for $h(x)$, in terms of x.

y

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.6		Source
(b)	2	1.2				2			1.2.6		1991 P1 qu.19

O
x
y

- $\bullet^1$ $p = 1$
- $\bullet^2$ $r = 46$
- $\bullet^3$ $h(x) = g(f(x))$
- $\bullet^4$ $h(x) = (2x+5)^2 - 3$

[SQA] 3. On a suitable set of real numbers, functions f and g are defined by $f(x) = \dfrac{1}{x+2}$

and $g(x) = \dfrac{1}{x} - 2$.

Find $f\big(g(x)\big)$ in its simplest form. **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
.	3	1.2	3						1.2.6		1992 P1 qu.6

$\bullet^1 \quad f\left(\dfrac{1}{x} - 2\right)$

$\bullet^2 \quad \dfrac{1}{\frac{1}{x} - 2 + 2}$

$\bullet^3 \quad x$

[SQA] 4. $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$.

(a) Find a formula for $k(x)$ where $k(x) = f\big(g(x)\big)$. **2**

(b) Find a formula for $h\big(k(x)\big)$. **2**

(c) What is the connection between the functions h and k? **1**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
(a)	2	1.2	2						1.2.6		
(b)	2	1.2	2						1.2.6		1993 P1 qu.13
(c)	1	0.1		1					0.1		

$\bullet^1 \quad f(3 - 2x)$

$\bullet^2 \quad 5 - 4x$

$\bullet^3 \quad h(5 - 4x)$

$\bullet^4 \quad x$

$\bullet^5 \quad$ inverse of each other

[SQA] 5. A function f is defined on the set of real numbers by $f(x) = \dfrac{x}{1-x}$, $x \neq 1$.

Find, in its simplest form, an expression for $f(f(x))$. **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
.	3	1.2	1	2					1.2.6		1994 P1 qu.19

- •^{1} $f\left(\frac{x}{1-x}\right)$

- •^{2} $\dfrac{\frac{x}{1-x}}{1-\frac{x}{1-x}}$

- •^{3} $\dfrac{x}{1-2x}$

[SQA] 6. The functions f and g, defined on suitable domains, are given by $f(x) = \dfrac{1}{x^2 - 4}$ and $g(x) = 2x + 1$.

(a) Find an expression for $h(x)$ where $h(x) = g(f(x))$. Give your answer as a single fraction. **3**

(b) State a suitable domain for h. **1**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
											Source
(a)	3	1.2	2	1					1.2.6		
(b)	1	1.2		1					1.2.1		1995 P1 qu.11

- •^{1} $g\left(\dfrac{1}{x^2-4}\right)$

- •^{2} $2\left(\dfrac{1}{x^2-4}\right)+1$

- •^{3} $\dfrac{x^2-2}{x^2-4}$

- •^{4} "any domain which excludes 2"

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7. Functions f and g, defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$.

Find $f(g(x))$ and $g(f(x))$. **4**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		**1.2**
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	4	1.2	4						1.2.6		1997 P1 qu.3

- •¹ $f(\sin x + \cos x)$
- •² $2(\sin x + \cos x)$
- •³ $g(2x)$
- •⁴ $\sin 2x + \cos 2x$

8. Functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = \dfrac{x^2 + 25}{x^2 - 25}$ where $x \in \mathbb{R}$, $x \neq \pm 5$.

The function h is given by the formula $h(x) = g(f(x))$.

For which real values of x is the function h **undefined**? **4**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		**1.2**
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
.	4	1.2					2	2	1.2.6	1.2.1	1989 P1 qu.19

- •¹ $g(2x+3)$
- •² $\dfrac{(2x+3)^2 + 25}{(2x+3)^2 - 25}$
- •³ $(2x+3)^2 - 25 = 0$
- •⁴ $x = 1, -4$

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[SQA] 9. The functions f and g are defined on a suitable domain by $f(x) = x^2 - 1$ and $g(x) = x^2 + 2$.

    (a) Find an expression for $f\big(g(x)\big)$.   **2**

    (b) Factorise $f\big(g(x)\big)$.   **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	1.2
(a)	2	1.2					2		1.2.6	Source
(b)	2	1.2					1	1	0.1	1998 P1 qu.6

OR

•¹ $f(x^2 + 2)$ •³ $\big((x^2+2)+1\big)\big((x^2+2)-1\big)$ •³ $x^4 + 4x^2 + 3$

•² $(x^2+2)^2 - 1$ •⁴ $(x^2+3)(x^2+1)$ •⁴ $(x^2+3)(x^2+1)$

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[SQA] 10. (a) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. (2)

(ii) Find $f(g(x))$. (2)

(b) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to
$2x^2 + 4x - k = 0$. (2)

(ii) Determine the nature of the roots of this equation when $k = 6$. (2)

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. (3)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(a)	4	1.2	4						1.2.6		1996 Paper 2
(b)	7	2.1	7						2.1.6,	2.1.7, 0.1	Qu.4

(a) $\bullet^1$ $g(2x+1)$

$\bullet^2$ $(2x+1)^2 + k$

$\bullet^3$ $f(x^2 + k)$

$\bullet^4$ $2(x^2 + k) + 1$

(b) $\bullet^5$ $4x^2 + 4x + k + 1$ **AND** $2x^2 + 2k + 1$

$\bullet^6$ $4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0$

so $2x^2 + 4x - k = 0$

$\bullet^7$ $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$

$\bullet^8$ so roots real & distinct

$\bullet^9$ $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$

$\bullet^{10}$ $b^2 - 4ac = 0$ for equal roots

$\bullet^{11}$ $k = -2$

[SQA] 11. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

    (*a*) Find formulae for

        (i) $f(g(x))$

        (ii) $g(f(x))$.   **4**

    (*b*) The function h is defined by $h(x) = f(g(x)) + g(f(x))$.

        Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h.   **3**

    (*c*) Find the area enclosed between this graph and the x-axis.   **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.2
(a)	4	1.2	4						1.2.6		Source 1999 Paper 2 Qu. 6
(b)	3	1.2	3						1.2.9	0.1	
(c)	4	2.2	4						2.2.6		

(a)   •^{1} $f(x^2)$ *stated or implied by* •^{2}

     •^{2} $x^2 - 1$

     •^{3} $g(x-1)$ *stated or implied by* •^{4}

     •^{4} $(x-1)^2$

(b)   •^{5} $(x-1)^2 + x^2 - 1$ *and complete proof*

     •^{6} *sketch as shown*

     •^{7} *minimum at* $\left(\frac{1}{2}, -\frac{1}{2}\right)$ *calculated or on sketch*

(c)   •^{8} $\int_0^1 \left(2x^2 - 2x\right) dx$

     •^{9} $\left[\frac{2}{3}x^3 - x^2\right]$

     •^{10} $-\frac{1}{3}$

     •^{11} *dealing with* $-$ve

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[SQA] 12. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

    (*a*) Find expressions for:

        (i) $f(h(x))$;

        (ii) $g(h(x))$. **2**

    (*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

        (ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x+\frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

$\bullet^1$ ic: interpret composite functions $\bullet^2$ ic: interpret composite functions $\bullet^3$ ss: expand $\sin(x + \frac{\pi}{4})$ $\bullet^4$ ic: interpret $\bullet^5$ ic: substitute $\bullet^6$ pd: start solving process $\bullet^7$ pd: process	$\bullet^1$ $\sin(x + \frac{\pi}{4})$ $\bullet^2$ $\cos(x + \frac{\pi}{4})$ $\bullet^3$ $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete** $\bullet^4$ $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$ $\bullet^5$ $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$ $\bullet^6$ $\frac{2}{\sqrt{2}}\sin x$ $\bullet^7$ $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 13. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

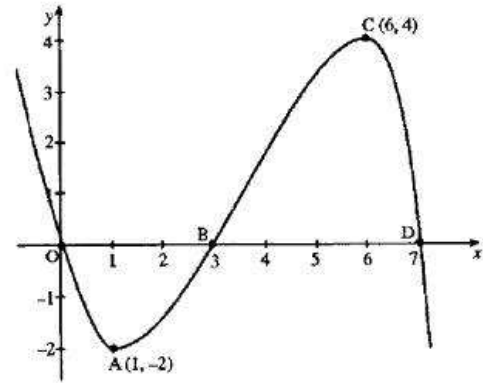 (*a*) Find expressions for:

   (i) $f(g(x))$;

   (ii) $g(f(x))$. **2**

  (*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \le x \le 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x^\circ)$, (ii) $2\sin(x^\circ)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$	

$\bullet^1$ ic: interpret $f(g(x))$ $\bullet^2$ ic: interpret $g(f(x))$ $\bullet^3$ ss: equate for intersection $\bullet^4$ ss: substitute for $\sin 2x$ $\bullet^5$ pd: extract a common factor $\bullet^6$ pd: solve a 'common factor' equation $\bullet^7$ pd: solve a 'linear' equation	$\bullet^1$ $\sin(2x^\circ)$ $\bullet^2$ $2\sin(x^\circ)$ $\bullet^3$ $2\sin(2x^\circ) = 2\sin(x^\circ)$ $\bullet^4$ appearance of $2\sin(x^\circ)\cos(x^\circ)$ $\bullet^5$ $2\sin(x^\circ)(2\cos(x^\circ) - 1)$ $\bullet^6$ $\sin(x^\circ) = 0$ **and** $0, 180, 360$ $\bullet^7$ $\cos(x^\circ) = \frac{1}{2}$ **and** $60, 300$ **or** $\bullet^6$ $\sin(x^\circ) = 0$ **and** $\cos(x^\circ) = \frac{1}{2}$ $\bullet^7$ $0, 60, 180, 300, 360$

[SQA]   14.   Part of the graph of $y = f(x)$ is shown in the diagram.
On separate diagrams sketch the graphs of

(a) $y = f(x+1)$

(b) $y = -2f(x)$.

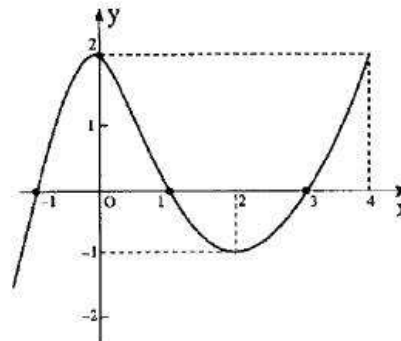Indicate on each graph the images of O, A, B, C and D.

1

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	1.2
			C	A/B	C	A/B	C	A/B		Source
(a)	2	1.2	2						1.2.4	1999 P1 qu.10
(b)	3	1.2	1	2					1.2.4	

•¹ translation of $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

•² positions of images of A, B, C, D, O clear from the sketch

•³ reflect in x - axis

•⁴ double y - coordinates

•⁵ positions of images of A, B, C, D, O clear from the sketch

[SQA]   15.   The diagram shows the graph of $y = f(x)$.
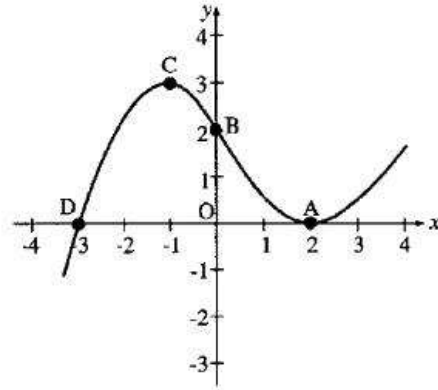Sketch the graph of $y = 2 - f(x)$.

3

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	1.2
			C	A/B	C	A/B	C	A/B		Source
.	3	1.2	1	2					1.2.4	1993 P1 qu.8

•¹ reflection in Ox

•² translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$

•³ two trans. in correct order, annotate diagram

[SQA] 16. Part of the graph of $y = f(x)$ is shown in the diagram.
On separate diagrams sketch the graphs of

(i) $y = f(x - 1)$

(ii) $y = -f(x) - 2$

indicating on each graph the images of A, B, C and D.

5

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		1.2
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(i)	2	1.2					2		1.2.4		1996 P1 qu.8
(ii)	3	1.2						3	1.2.4		

•¹ translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

•⁵ $A(2, -2)$ $B(0, -4)$ $C(-1, -5)$ $D(-3, -2)$

•² $A(3, 0)$ $B(1, 2)$ $C(0, 3)$ $D(-2, 0)$

•³ reflect in x − axis

•⁴ translation of $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$

[END OF WRITTEN QUESTIONS]