X100/12/11

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may <u>NOT</u> be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

Section B (30 marks)

Full credit will be given only where the solution contains appropriate working.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers clearly in the spaces in the answer booklet provided. Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

Read the questions for **Section A** and record your answers on the grid on Page three of the answer booklet.

1. The answer to each question is **either** A, B, C or D. Decide what your answer is, then fill in the appropriate bubble (see sample question below).

2. There is **only one correct** answer to each question.

Do NOT use gel pens.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

A   1

B   8

C   0

D   –4

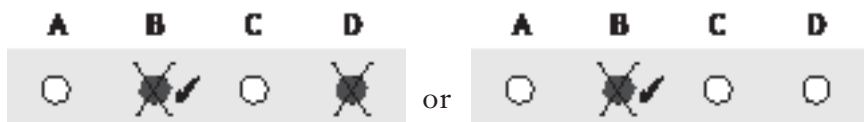The correct answer is **B**—8. The answer **B** bubble has been clearly filled in (see below).

Changing an answer

If you decide to change your answer, cancel your first answer by putting a cross through it (see below) and fill in the answer you want. The answer below has been changed to **D**.

If you then decide to change back to an answer you have already scored out, put a tick (✓) to the **right** of the answer you want, as shown below:

or

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\quad \mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

$\qquad$ or $\qquad \mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

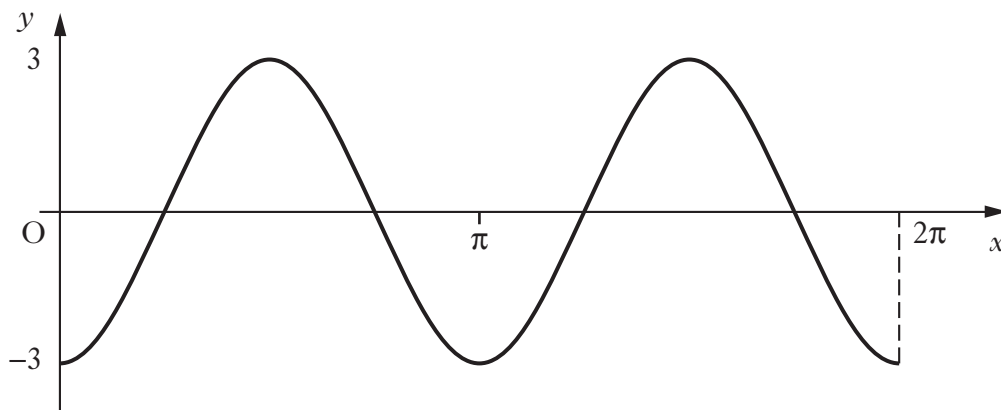| $f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + c$
$\cos ax$	$\dfrac{1}{a} \sin ax + c$

[Turn over

1. Given $f(x) = 2x^3 - 7$ find the value of $f'(2)$.

   A -6

   B   9

   C   24

   D   137

2. The line with equation $2y = 3x + 5$ is perpendicular to the line with equation $y = kx$.
   What is the value of k?

   A $-\dfrac{3}{2}$

   B $-\dfrac{2}{3}$

   C $\dfrac{2}{3}$

   D $\dfrac{3}{2}$

3. If $2x^3 + x^2 - 4x + 1$ is divided by $(x - 2)$, what is the remainder?

   A -11

   B   0

   C   1

   D   13

4. The diagram shows the graph with equation of the form $y = a\cos bx$ for $0 \le x \le 2\pi$.

What is the equation of this graph?

A $y = -3\cos 2x$

B $y = -3\cos 3x$

C $y = 3\cos 2x$

D $y = 3\cos 3x$

5. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}2u_n + 9$, $u_5 = 11$.
What is the value of u_3?

A $11{\cdot}24$

B $9{\cdot}4$

C   5

D   4

6. The points P, Q and R are collinear.

P is the point $(-1, 6, 4)$, Q is the point $(2, 0, 13)$ and $\overrightarrow{QR} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$.

Calculate the ratio in which Q divides PR.

A $2 : 3$

B $3 : 2$

C $3 : 5$

D $5 : 2$

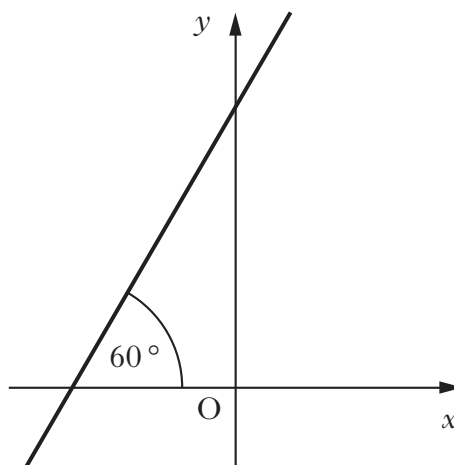**7.** What is $\int (x+4)(x-4)\,dx$?

A $2x + c$

B $\frac{1}{3}x^3 + c$

C $\frac{1}{3}x^3 - 16x + c$

D $\left(\frac{1}{2}x^2 + 4x\right)\left(\frac{1}{2}x^2 - 4x\right) + c$

8. A straight line makes an angle of $60°$ with the x-axis as shown in the diagram.

What is the gradient of this line?

A $\frac{1}{2}$

B $\frac{1}{\sqrt{3}}$

C $\frac{\sqrt{3}}{2}$

D $\sqrt{3}$

9. Find the minimum value of $3\sin2x + 5$ and the value of x where this occurs in the interval $0 \leq x < \pi$.

	min value	x
A	2	$\dfrac{3\pi}{2}$
B	2	$\dfrac{3\pi}{4}$
C	4	$\dfrac{3\pi}{2}$
D	4	$\dfrac{3\pi}{4}$

10. Solve $2\cos x + 1 = 0$ for x, where $\pi \leq x \leq \dfrac{3\pi}{2}$.

A $\dfrac{5\pi}{6}$

B $\dfrac{7\pi}{6}$

C $\dfrac{5\pi}{4}$

D $\dfrac{4\pi}{3}$

11. The curve $y = f(x)$ is such that $f'(x) = 4x - 1$.

The curve passes through the point $(2, 9)$.

What is the equation of the curve?

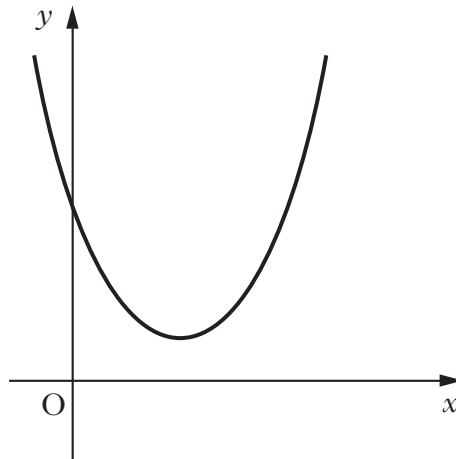A $y = 2x^2 - x - 5$

B $y = 2x^2 + 1$

C $y = 2x^2 - x + 3$

D $y = 2x^2 - x$

[Turn over

12. Given that the point R is $(3, -1, 2)$, $\overrightarrow{RS} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\overrightarrow{RT} = 3\overrightarrow{RS}$,

find the coordinates of T.

A $(3, 2, -11)$

B $(3, 4, -11)$

C $(9, 2, -7)$

D $(9, 4, -7)$

13. The diagram shows a curve with equation of the form $y = ax^2 + bx + c$.

Here are two statements about a, b and c:

    (1) $a > 0$

    (2) $b^2 - 4ac > 0$

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

14. If $\cos x = -\dfrac{2}{5}$, what is the value of $\cos 2x$?

A $\dfrac{33}{25}$
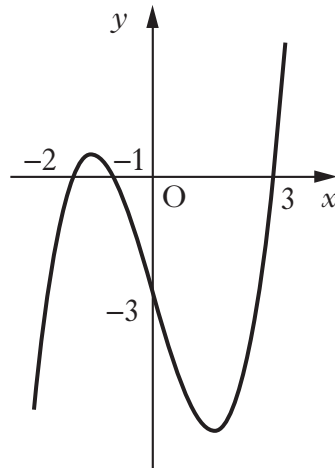
B $\dfrac{17}{25}$

C $-\dfrac{4}{5}$

D $-\dfrac{17}{25}$

15. The graph of a cubic function, $y = f(x)$, is shown below.

It passes through the points $(-2, 0)$, $(-1, 0)$, $(3, 0)$ and $(0, -3)$.

What is the equation of this curve?

A $y = \frac{1}{2}(x-3)(x+1)(x+2)$

B $y = 2(x-3)(x+1)(x+2)$

C $y = -\frac{1}{2}(x+3)(x-1)(x-2)$

D $y = -2(x+3)(x-1)(x-2)$

[Turn over

16. If $e^{4t} = 6$, find an expression for t.

A $\quad t = \log_e \dfrac{3}{2}$

B $\quad t = \dfrac{\log_e 6}{4}$

C $\quad t = \dfrac{6}{\log_e 4}$

D $\quad t = \dfrac{\log_e 6}{\log_e 4}$

17. Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} \frac{3}{5} \\ 0 \\ t \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 0 \\ -10 \end{pmatrix}$ respectively.

Here are two statements about $\mathbf{u}$ and $\mathbf{v}$:

(1) $\quad$ when $t = \dfrac{4}{5}$, $\mathbf{u}$ is a unit vector

(2) $\quad$ when $t = 1$, $\mathbf{u}$ and $\mathbf{v}$ are parallel

Which of the following is true?

A $\quad$ Neither statement is correct.

B $\quad$ Only statement (1) is correct.

C $\quad$ Only statement (2) is correct.

D $\quad$ Both statements are correct.

18. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k?
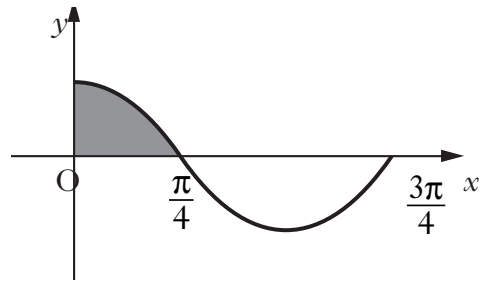
A $\quad$ 5

B $\quad$ 6

C $\quad$ 25

D $\quad$ 36

19. The diagram shows part of the graph of $y = a \cos bx$.

The shaded area is $\frac{1}{2}$ unit^2.

What is the value of $\int_0^{\frac{3\pi}{4}} (a\cos bx)\, dx$?

A -1

B $-\frac{1}{2}$

C $\frac{1}{2}$

D $1\frac{1}{2}$

20. The only stationary point on the graph of $y = f(x)$ is the point (a, b).

What are the coordinates of the only stationary point on the graph of $y = -f(2x)$?

A $\left(\frac{1}{2}a, -b\right)$

B $(2a, -b)$

C $\left(-\frac{1}{2}a, b\right)$

D $(-2a, b)$

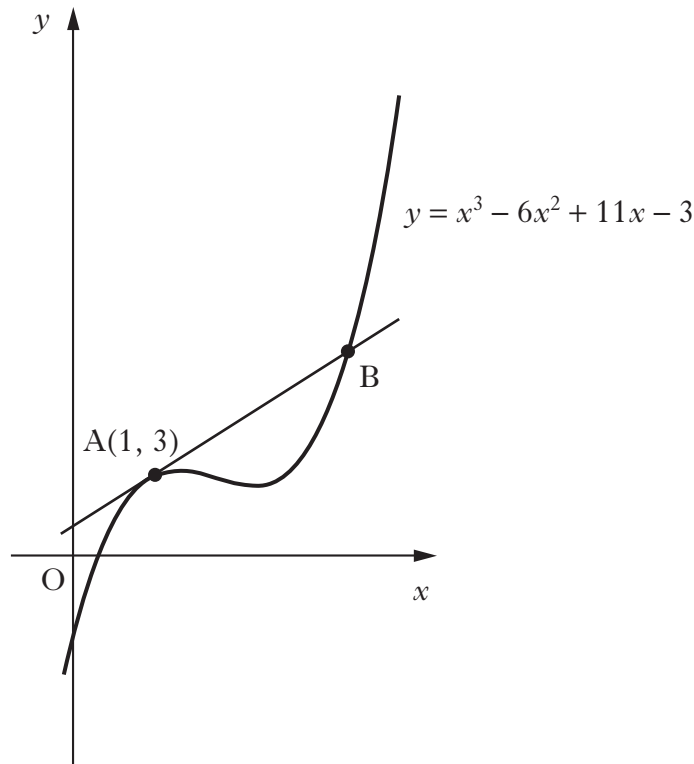[*END OF SECTION A*]

[Turn over

ALL questions should be attempted.

21. (*a*) Show that $(x - 1)$ is a factor of $x^3 - 6x^2 + 9x - 4$ and hence
factorise $x^3 - 6x^2 + 9x - 4$ fully.   **4**

(*b*) The diagram shows the graph with equation $y = x^3 - 6x^2 + 11x - 3$.

(i) Find the equation of the tangent to the curve $y = x^3 - 6x^2 + 11x - 3$ at the
point A(1, 3).   **3**

(ii) Hence find the coordinates of B, the point of intersection of this tangent
with the curve.   **3**

22. The function $f(x) = \dfrac{4}{x^2} + x$ is defined on the domain $x > 0$, $x \in \mathbb{R}$, the set of real numbers.

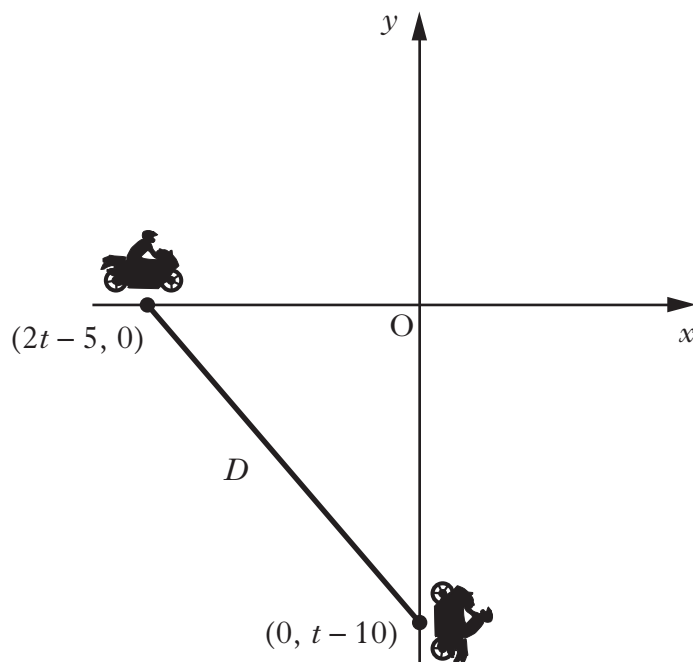Find the maximum and minimum values of $f(x)$ on the closed interval $1 \leq x \leq 4$.   **6**

23. Solve $\log_2(3x + 7) = 3 + \log_2(x - 1)$, $x > 1$.   **4**

24. Find the range of values for k such that $kx^2 + 3x + 9k = 0$ has real roots.   **4**

25. A stunt performed by two members of a motorcycle display team requires them to travel, at speed, at right angles to each other across the arena.

The positions of the motorcyclists, relative to suitable axes, t seconds after the stunt begins, are $(2t - 5, 0)$ and $(0, t - 10)$.

(*a*) Show that, at any given moment, the distance, D, between them is given by

$$D = \sqrt{5t^2 - 40t + 125}.$$ **2**

(*b*) Determine whether the distance between the motorcyclists is increasing or decreasing 5 seconds after the start of the stunt.   **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

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X100/12/13

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or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

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$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
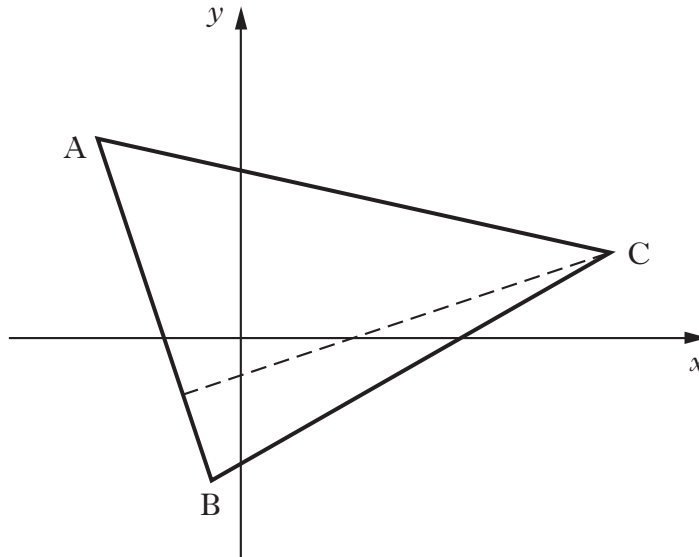$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + c$
$\cos ax$	$\dfrac{1}{a} \sin ax + c$

1. The vertices of triangle ABC are A(–5, 7), B(–1, –5) and C(13, 3) as shown in the diagram.

The broken line represents the altitude from C.

(*a*) Show that the equation of the altitude from C is $x - 3y = 4$.   **4**

(*b*) Find the equation of the median from B.   **3**

(*c*) Find the coordinates of the point of intersection of the altitude from C and the median from B.   **2**

2. Functions f and g are defined on suitable domains by

$$f(x) = 10 + x \text{ and } g(x) = (1 + x)(3 - x) + 2.$$

(*a*) Find an expression for $f(g(x))$.   **2**

(*b*) Express $f(g(x))$ in the form $p(x + q)^2 + r$.   **3**

(*c*) Another function h is given by $h(x) = \dfrac{1}{f(g(x))}$.

What values of x cannot be in the domain of h?   **2**

[Turn over

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32, \quad f_1 = 32$

- $t_{n+1} = \frac{3}{4}t_n + 13, \quad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(*a*) Calculate t_2, the height of the toad at the end of the second day. **1**

(*b*) Determine whether or not either of them will eventually escape from the well. **5**

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.
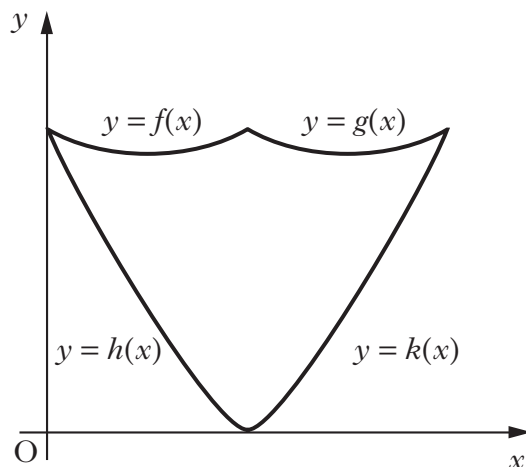
- $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$

- $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

- $h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3$

- $k(x) = \frac{3}{8}x^2 - \frac{3}{4}x$

(*a*) Find the *x*-coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$.   **2**

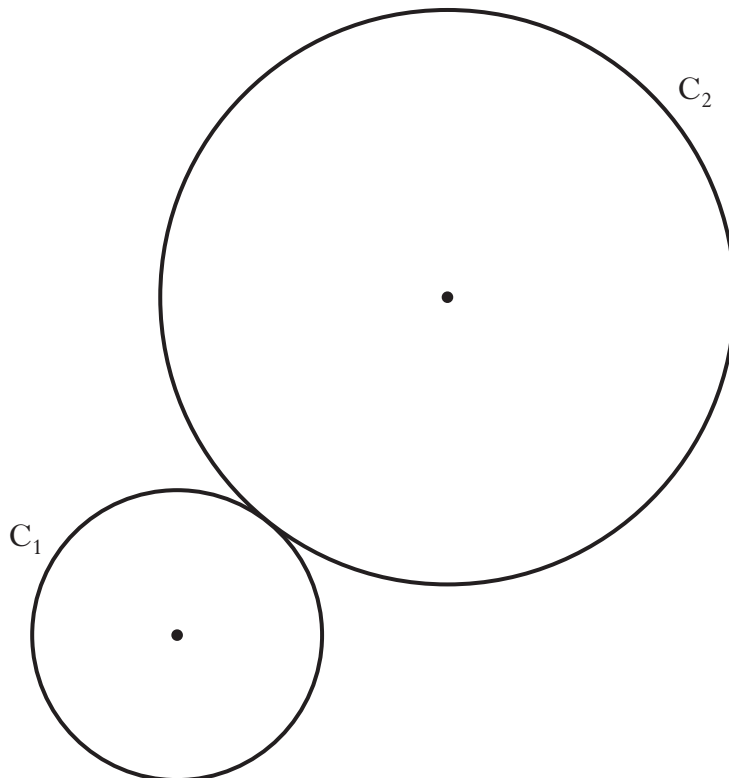The graphs of the functions $f(x)$ and $h(x)$ intersect on the *y*-axis.

The plaque has a vertical line of symmetry.

(*b*) Calculate the area of the wall plaque.   **7**

[Turn over

5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$.

The centre of circle C_2 is (9, 11).

Circles C_1 and C_2 touch externally.

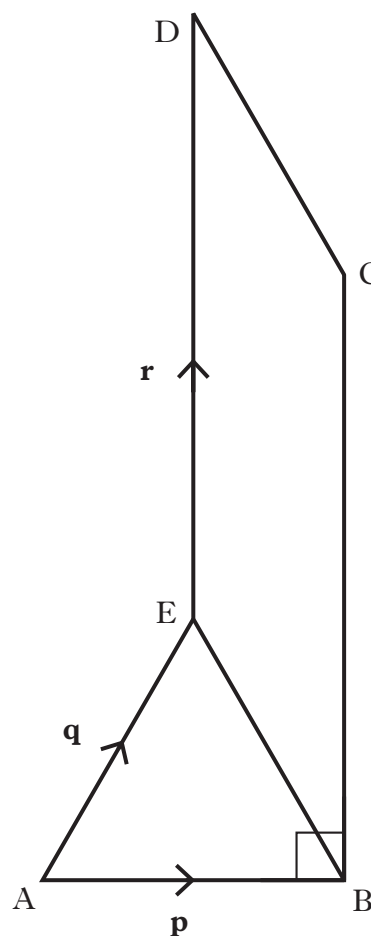(*a*) Determine the radius of C_2. **4**

A third circle, C_3, is drawn such that:

- both C_1 and C_2 touch C_3 internally
- the centres of C_1, C_2 and C_3 are collinear.

(*b*) Determine the equation of C_3. **4**

6. Vectors **p**, **q** and **r** are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC = 90°

(*a*) Evaluate **p.(q + r)**.   **3**

(*b*) Express $\overrightarrow{EC}$ in terms of **p**, **q** and **r**.   **1**

(*c*) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \dfrac{9}{2}$, find $|\mathbf{r}|$.   **3**

[Turn over for Questions seven and eight on *Page eight*

7. (*a*) Find $\int (3\cos 2x + 1)\,dx$.   **2**

    (*b*) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$.   **2**

    (*c*) Hence, or otherwise, find $\int (\sin^2 x - 2\cos^2 x)\,dx$.   **2**

8. The blades of a wind turbine are turning at a steady rate.

The height, *h* metres, of the tip of one of the blades above the ground at time, *t* seconds, is given by the formula

$$h = 36\sin(1\cdot5t) - 15\cos(1\cdot5t) + 65.$$

Express $36\sin(1\cdot5t) - 15\cos(1\cdot5t)$ in the form

$$k\sin(1\cdot5t - a), \text{ where } k > 0 \text{ and } 0 < a < \frac{\pi}{2},$$

and hence find the **two** values of *t* for which the tip of this blade is at a height of 100 metres above the ground during the first turn.   **8**

[END OF QUESTION PAPER]