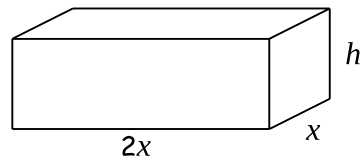
Higher Homework 12

1) Solve,

a) $\cos 2x - 2\sin^2 x = 0$ for $0 \le x \le 360°$.

b) $\cos 2x - 2\sin^2 x = 0$ for $0 \le x \le 2\pi$.

2) An open cuboid measures x units by $2x$ units by h units and has an inner surface area of 12 units^2.



a) Show that the volume, V units^3, of the cuboid is given by

$$V(x) = \frac{2}{3}x(6 - x^2).$$

b) Find the exact value of x for which this volume is a maximum.

3) a) Write $x^2 - 10x + 27$ in the form $(x-b)^2 + c$.

b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

4) With reference to a suitable set of coordinate axes, A, B and C are the points (-8, 10, 20), (-2, 1, 8) and (0, -2, 4) respectively.

Show that A, B and C are collinear and find the ratio AB : BC.

5) P, Q and R have coordinates (1, 3, -1) (2, 0, 1) and (-3, 1, 2) respectively.

a) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form.

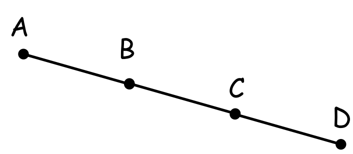
b) Hence or otherwise find the size of the angle PQR.

6) Solve,

a) $\sin 2x = 6\cos x$ for $0 \le x \le 360°$.

b) $\sin x - \sin 2x = 0$ for $0 \le x \le 360°$.

7) A and B are the points (-1, -3, 2) and (2,-1, 1) respectively.
B and C are the points of trisection of AD, that is
AB = BC = CD.



Find the coordinates of D.

8) Given that $(x - 2)$ and $(x - 3)$ are factors of $f(x)$ where $f(x) = 3x^3 + 2x^2 + cx + d$, find the values of c and d.

9) Find the positive value of z for which

$$\int_2^z (6x - 5)\, dx = 10$$