

Higher HW4

1a. $m_{AB} = \frac{1-5}{3+1}$
 $= \frac{-4}{4}$
 $= -1$

$y-1 = -1(x-3)$
 $y-1 = -x+3$
 $y = -x+4$

b. $m = \tan \theta$
 $\tan \theta = -1$
 $\theta = \tan^{-1}(-1)$
 $= 135^\circ$

$\frac{S}{T} \mid A 45^\circ$
 $\frac{S}{T} \mid C$
 $180 - 45 = 135^\circ$

2. $U_{n+1} = aU_n + 10$
 $L = \frac{10}{1-a}$

$V_{n+1} = a^2V_n + 16$
 $L = \frac{16}{1-a^2}$

$\frac{16}{1-a^2} = \frac{10}{1-a}$
 $16(1-a) = 10(1-a^2)$
 $16-16a = 10-10a^2$
 $10a^2-16a+6 = 0$
 $5a^2-8a+3 = 0$
 $(5a-3)(a-1) = 0$
 $a = \frac{3}{5}$ or $a = 1$

As both sequences have a limit,
 $a = \frac{3}{5}$
 $L = \frac{10}{\frac{2}{5}}$
 $= 10 \times \frac{5}{2}$
 $= 25$

3a) $f(g(x))$
 $= f(\frac{1}{5}x+3)$
 $= 5(\frac{1}{5}x+3)^2 - 2$
 $= 5(\frac{1}{25}x^2 + \frac{6}{5}x + 9) - 2$
 $= \frac{1}{5}x^2 + 6x + 43$

ii) $g(f(x))$
 $= g(5x^2-2)$
 $= \frac{1}{5}(5x^2-2) + 3$
 $= x^2 - \frac{2}{5} + 3$
 $= x^2 + \frac{13}{5}$

b) $f(x) = 5x^2 - 2$
 $y = 5x^2 - 2$
 $y+2 = 5x^2$
 $\frac{y+2}{5} = x^2$
 $x = \sqrt{\frac{y+2}{5}}$
 $f^{-1}(x) = \sqrt{\frac{x+2}{5}}$

ii) $g(x) = \frac{1}{5}x + 3$
 $y = \frac{1}{5}x + 3$
 $y-3 = \frac{1}{5}x$
 $5(y-3) = x$
 $x = 5(y-3)$
 $g^{-1}(x) = 5(x-3)$

4. $U_{n+1} = aU_n + b$
 $U_7 = 8a + b = 21$ ①
 $U_8 = 21a + b = 47$ ②
 \times ① by -1
 $-8a - b = -21$
 $21a + b = 47$
 $13a = 26$
 $a = 2$

Place $a=2$ into ①
 $16 + b = 21$
 $b = 5$

$$5. U_{n+1} = 0.8U_n + 5$$

$$L = \frac{5}{1-0.8}$$

$$= \frac{5}{0.2}$$

$$= \frac{50}{2}$$

$$= 25$$

The pressure will settle at around 25 units

$$6a. U_{n+1} = 1.015U_n - 300$$

b.

$$\text{1st March} = 2500$$

$$\text{April} = 2237.50$$

$$\text{May} = 1971.06$$

$$\text{June} = 1700.63$$

$$\text{July} = 1426.14$$

$$\text{Aug} = 1147.53$$

$$\text{Sep} = 864.74$$

$$\text{Oct} = 577.71$$

$$\text{Nov} = 286.37$$

$$\text{1st Dec} = \underline{\underline{£290.68}}$$

Final payment of $\underline{\underline{£290.68}}$ on the 1st of December

$$7a. 3x^2 + 11x - 20 = 0$$

$$(3x-4)(x+5) = 0$$

$$x = \frac{4}{3} \text{ or } x = -5$$

$$\left(\frac{4}{3}, 0\right) \quad (-5, 0)$$

$$b. \frac{4}{3} + \frac{15}{3} = \frac{19}{3}$$

$$\frac{19}{3} \div 2 = \frac{19}{6}$$

$$\frac{8-19}{6} = \frac{-11}{6}$$

$$\left(\frac{11}{6}, \frac{41}{4}\right)$$

$$3x\left(\frac{11}{6}\right)^2 + 11\left(\frac{11}{6}\right) - 20$$

$$= \frac{41}{4}$$

$$\left(\frac{11}{6}, \frac{41}{4}\right)$$

$$c. \text{Roots: } \left(\frac{2}{3}, 0\right), (-7, 0)$$

$$\text{TP: } \left(\frac{7}{6}, \frac{41}{6}\right)$$

$$d. \text{Roots: } \left(\frac{4}{15}, 0\right), (-1, 0)$$

$$\text{TP: } \left(\frac{11}{30}, \frac{41}{4}\right)$$

$$8a. x^2 + 4x + 5$$

$$= \underline{\underline{(x+2)^2 + 1}}$$

$$b. x^2 - 12x - 13$$

$$= \underline{\underline{(x-6)^2 - 49}}$$