Higher Homework 8

Section 1: L.R.R.

1. For each of the L.R.R.'s below, state, with justification, whether there is a limit.
When the limit exists, determine it algebraically; when there is no limit, find the least value of n such that u_n exceeds 100.

(a) $u_{n+1} = 0 \cdot 8u_n + 10 \quad (u_0 = 10)$

(b) $u_{n+1} = 2u_n - 5 \quad (u_0 = 6)$

(c) $u_{n+1} = -0 \cdot 9u_n + 3 \cdot 8 \quad (u_0 = 6)$

(d) $u_{n+1} = 1 \cdot 5u_n + 10 \quad (u_0 = 1)$

2. A microbiologist has noticed that 20% of her cell cultures become infected and die each day.
She can prepare 80 new cultures at the start of each day.
If she starts day 1 with 80 cultures,

(a) How many cultures will there be at the start of days 2, 3, 4 and 5 ? (Just after the 80 new cultures are added.)

(b) Find a formula that models this situation.

(c) What is the 'long term' situation?

3. (a) At 12 noon a patient is given a pill containing 50 units of antibiotic.
By 1 p.m. the number of units in his body has dropped by 12%.
By 2 p.m. a further 12% of the units remaining in his body at 1 p.m. is lost.
If this fall off rate is maintained, find the number of units of antibiotic remaining at 6 p.m.

(b) A doctor considers prescribing a course of treatment which involves the patient taking one of these pills every 6 hours over a long period of time.
More than 100 units of this antibiotic in the body is considered dangerous.
Should the doctor prescribe this treatment? Give reasons.

Section 2: Straight Lines

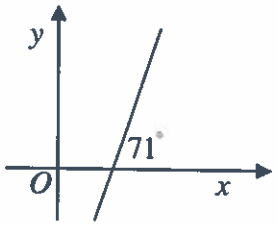
1. (a) Sketch the graph of each of these straight lines:

(i) $y = 2x - 3$ (ii) $2x + 3y = 6$

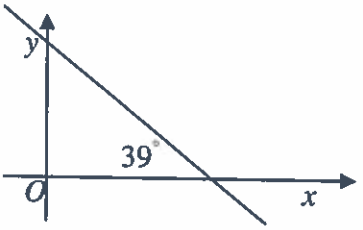
(b) Calculate the size of the angle each line, above, makes with the positive direction of the x-axis.

2. Calculate the gradient of each of these straight lines:

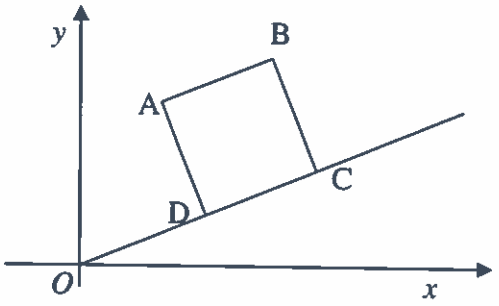
(a)



(b)



3.



The straight line ODC has equation $y = \frac{1}{2}x$. A is the point (3,4). ABCD is a square.

(a) Find the equation of the straight line AD.
(b) Find the coordinates of D.
(c) Find the area of square ABCD.

Section 3: Differential Calculus

1. $y = x^3 + x^2 - 16x - 16$.

Find the coordinates of the stationary points and determine their nature. Justify your answer.

2. Find the equation of the tangent to the curve $y = 2x^3 - 2$ at the point where $x = -1$.

3. For which values of x is the function $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x$ decreasing?

4. $f(x) = (x-1)^2(x+2)$ $(x \in R)$.

(a) Find the coordinates of the points where the curve with equation $y = f(x)$ meets the coordinate axes.

(b) Find the stationary points and determine their nature.

(c) Sketch the curve.