Higher HW8

SECTION 1

1) a) $u_{n+1} = 0.8u_n + 10$ $(u_0 = 10)$

Limit exists because $-1 < 0.8 < 1$

$L = \frac{10}{1-0.8} = \frac{10}{0.2} = 50$

b) $u_{n+1} = 2u_n - 5$ $(u_0 = 6)$

No Limit exist because $-1 \not< 2 \not< 1$

$u_1 = 7$, $u_2 = 9$, $u_3 = 13$, $u_4 = 21$, $u_5 = 37$, $u_6 = 69$, $u_7 = 133$

u_n first exceeds 100 when $n = 7$.

c) $u_{n+1} = -0.9u_n + 3.8$ $(u_0 = 6)$

Limit exists because $-1 < -0.9 < 1$

$L = \frac{3.8}{1+0.9} = \frac{3.8}{1.9} = 2$

d) $u_{n+1} = 1.5u_n + 10$ $(u_0 = 1)$

No limit exists because $-1 \not< 1.5 \not< 1$

$u_1 = 11.5$, $u_2 = 27.25$, $u_3 = 50.875$, $u_4 = 86.312...$, $u_5 = 139.46...$

u_n first exceeds 100 when $n = 5$

2) 20% die (80% remain) $\rightarrow \times 0.8$ $(+80)$ $u_1 = 80$

a) $u_2 = 144$ cultures

$u_3 = 195.2$ cultures

$u_4 = 236.16$ cultures

$u_5 = 268.928$ cultures

b) $u_{n+1} = 0.8u_n + 80$

c) Long term the no. of cultures will converge because $-1 < 0.8 < 1$

$\therefore L = \frac{80}{1-0.8} = \frac{80}{0.2} = 400$ cultures

3) a) $u_0 = 50$ units

$u_1 = 50 \times 0.88 = 44$

$u_2 = 44 \times 0.88 = 38.72$

$u_3 = 34.07...$

$u_4 = 29.98...$

$u_5 = 26.38...$

$u_6 = 23.2202...$

At 6pm there will be 23.22 units remaining (2dp)

b) Over 6hrs 0.88^6 gives decrease

$u_{n+1} = 0.88^6 u_n + 50$

A limit exists because $-1 < 0.88^6 < 1$

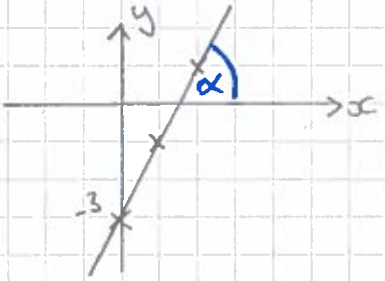
$L = \frac{50}{1-0.88^6} = 93.35$ units in blood as $t \rightarrow \infty$

$\therefore$ because $93.35 < 100$ units so the levels will be safe long term

SECTION 2

1) a) i) $y = 2x - 3$

$m = 2 \quad c = -3$

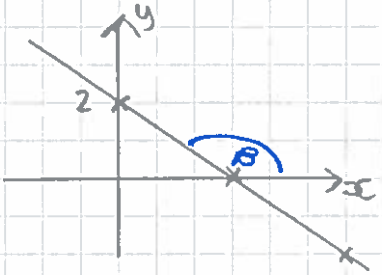


ii) $2x + 3y = 6$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

$m = -\frac{2}{3} \quad c = 2$



b) i) $\tan\theta = 2$

$\theta = \tan^{-1} 2$

$= 63.4349\ldots$

$= 63.43°$ (2dp)

ii) $\tan\theta = -2/3$

$\theta = \tan^{-1}(-2/3)$

$= 146.3099\ldots$

$= 146.31°$ (2dp)

2) a) $\tan 71 = 2.9047\ldots$

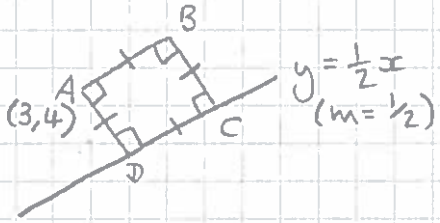
$= 2.9$ (1dp)

b) $180 - 39 = 141$

$\tan 141 = -0.8097\ldots$

$= -0.81$ (2dp)

3)



a) AD $\quad A(3,4)$

$m_{AD} = -2$ (perp to ½)

$y - 4 = -2(x - 3)$

$y - 4 = -2x + 6$

$y + 2x - 10 = 0$ (2)

b) $y = \frac{1}{2}x$

$2y = x$

$2y - x = 0$ (1)

(1) $2y - x + 0 = 0$ (+)

$-2 \times$ (2) $-2y - 4x + 20 = 0$

$-5x + 20 = 0$

$5x = 20$

$x = 4$

Input $x = 4$ into (1)

$2y - 4 = 0$

$y = 2$

D(4, 2)

c) Length AD

$= \sqrt{(4-3)^2 + (2-4)^2}$

$= \sqrt{1 + 4} = \sqrt{5}$ units

Area of ABCD

$= \sqrt{5} \times \sqrt{5}$

$= 5$ units^2

SECTION 3

1) $y = x^3 + x^2 - 16x - 16$

SP's exist when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 + 2x - 16$

$3x^2 + 2x - 16 = 0$

$(3x + 8)(x - 2) = 0$

$x = -\frac{8}{3}$ $\qquad$ $x = 2$

$y = \frac{400}{27}$ $\qquad$ $y = -36$

$(-\frac{8}{3}, \frac{400}{27})$ $\qquad$ $(2, -36)$

a falling point of inflexion $\qquad$ Min Turning Point

x	-2	$-\frac{8}{3}$	0	2	3
$\frac{dy}{dx}$	-8	0	-16	0	17
	\	−	\	_	/

2) $y = 2x^3 - 2$

$\frac{dy}{dx} = 6x^2$

m at $x = -1$

$m = 6 \times 1 = 6$

$x = -1 \rightarrow y = -2 - 2 = -4$

$(-1, -4)$

$y + 4 = 6(x + 1)$

$y + 4 = 6x + 6$

$y - 6x - 2 = 0$

3) $f(x) = \frac{1}{3}x^3 - 2x^2 - 5x$

is decreasing when $f'(x) < 0$

$f'(x) = x^2 - 4x - 5$

$= (x - 5)(x + 1)$

$f(x)$ is decreasing when

$-1 < x < 5$

4) $f(x) = (x-1)^2(x+2)$ $\quad x \in \mathbb{R}$

a) y axis $(x=0)$ $\quad f(0) = (-1)^2 \times 2 = 2$

$(0,2)$

x axis $(f(x)=0)$ $\quad (x-1)^2(x+2) = 0$

$x = 1, \; x = -2$

$(1,0)$ & $(-2,0)$

b) SPs exist when $f'(x) = 0$

$f(x) = (x^2 - 2x + 1)(x+2)$

$f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0$

$x = -1$ $\quad x = 1$

$y = 4$ $\quad y = 0$

Max TP $\quad$ Min TP

x	-2	-1	0	1	2
$f'(x)$	9	0	-3	0	9
	/	–	\	–	/

c)