Integration Past Papers Unit 2 Outcome 2

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Evaluate $\int_1^4 x^{-1/2}\,dx$.

   A. -2

   B. $-\frac{7}{16}$

   C. $\frac{1}{2}$

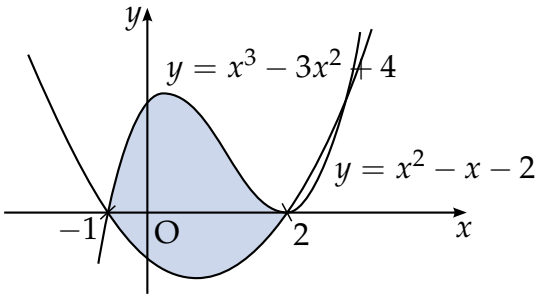
   D. 2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.2	C	0.71	0.64	NC	C13	HSN 086

$$\int_1^4 x^{-1/2}\,dx = \left[\frac{x^{1/2}}{1/2}\right]_1^4$$
$$= \left[2\sqrt{x}\right]_1^4$$
$$= 2\sqrt{4} - 2\sqrt{1}$$
$$= 4 - 2$$
$$= 2.$$

Option D

2. The diagram shows the area bounded by the curves $y = x^3 - 3x^2 + 4$ and $y = x^2 - x - 2$ between $x = -1$ and $x = 2$.



Represent the shaded area as an integral.

A. $\int_{-1}^{2} (x^3 - 4x^2 + x + 6)\, dx$

B. $\int_{-1}^{2} (-x^3 + 4x^2 - x - 6)\, dx$

C. $\int_{-1}^{2} (x^3 - 4x^2 - x + 2)\, dx$

D. $\int_{-1}^{2} (x^3 - 2x^2 - x + 2)\, dx$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
A	2.2	C	0.7	0.46	NC	C17	HSN 094

The shaded area is given by

$$\int_{-1}^{2} (\text{upper} - \text{lower})\, dx$$

$$= \int_{-1}^{2} \left(x^3 - 3x^2 + 4 - (x^2 - x - 2) \right) dx$$

$$= \int_{-1}^{2} \left(x^3 - 4x^2 + x + 6 \right) dx.$$

Option A

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA] 3. Find $\int \left(2x^2 + 3\right) dx$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
.	3	2.2	3						2.2.4		1989 P1 qu.5

•1 $\frac{2}{3}x^3$
•2 $+3x$
•3 $+c$

[SQA] 4. Find $\int \left(3x^3 + 4x\right) dx$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
.	3	2.2	3						2.2.4		1994 P1 qu.1

•1 $\frac{3}{4}x^4$
•2 $2x^2$
•3 $+c$

[SQA] 5. Evaluate $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
.	5	2.2	5						2.2.4		1998 P1 qu.12

•1 know to expand brackets
•2 $x^4 + 2x + x^{-2}$
•3 $\frac{1}{5}x^5 + x^2$
•4 $-\frac{1}{x}$
•5 $9\frac{7}{10}$

[SQA] 6. Find $\int \frac{x^2-5}{x\sqrt{x}}\,dx$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	4	2.2	2	2					2.2.4		Source: 1999 P1 qu.20

- •1 $\left(\frac{x^2}{x\sqrt{x}}=\right)\ x^{\frac{1}{2}}$
- •2 $\left(\frac{-5}{x\sqrt{x}}=\right)\ -5x^{-\frac{3}{2}}$
- •3 $\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$
- •4 $\frac{-5}{-\frac{1}{2}}x^{-\frac{1}{2}}$

[SQA] 7. Find $\int \frac{(x^2-2)(x^2+2)}{x^2}\,dx,\ x\neq 0$. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	4	C	CN	C14, C12, C13	$\frac{1}{3}x^3+4x^{-1}+c$	2001 P2 Q6

- •1 ss: start to write in standard form
- •2 pd: complete process
- •3 pd: integrate
- •4 pd: integrate a −ve index

- •1 $\frac{x^4-4}{x^2}$
- •2 x^2-4x^{-2}
- •3 $\frac{1}{3}x^3+c$
- •4 $\frac{-4x^{-1}}{-1}$

[SQA] 8. Find the value of $\int_1^2 \frac{u^2+2}{2u^2}\,du$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	5	2.2	4	1					2.2.5		Source: 1989 P1 qu.16

- •1 *strat*: know to divide
- •2 $\frac{1}{2}+u^{-2}$
- •3 $\frac{1}{2}u$
- •4 $-u^{-1}$
- •5 1

[SQA] 9.

(*a*) Find the value of $\int_1^2 \left(4 - x^2\right) dx$. **3**

(*b*) Sketch a graph and shade the area represented by the integral in (*a*). **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
(a)	3	2.2	3						2.2.5		1991 P1 qu.16
(b)	2	2.2	2						2.2.6		

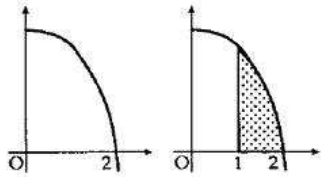
•1 $4x$

•2 $\frac{1}{3}x^3$

•3 $1\frac{2}{3}$

•4 for diagram 1 as shown

•5 shading 1 *to* 2



[SQA] 10. Evaluate $\int_1^9 \frac{x+1}{\sqrt{x}}\, dx$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
.	5	2.2	5						2.2.5		1992 P1 qu.8

•1 $x^{\frac{1}{2}}$

•2 $x^{-\frac{1}{2}}$

•3 $\frac{2}{3}x^{\frac{3}{2}}$

•4 $2x^{\frac{1}{2}}$

•5 $21\frac{1}{3}$

[SQA] 11. Find the value of $\int_1^4 \sqrt{x}\, dx$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2 Source
.	4	2.2	4						2.2.5		1997 P1 qu.10

•1 $x^{\frac{1}{2}}$

•2 $x^{\frac{3}{2}+\frac{3}{2}}$

•3 $\frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right)$

•4 $\frac{14}{3}$

[SQA] 12. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7-3x)}+c$	2000 P2 Q10

•1 pd: integrate function •2 pd: deal with function of function	•1 $\frac{1}{-1}(7-3x)^{-1}$ •2 $\times\frac{1}{-3}$

[SQA] 13. Evaluate $\int_{-3}^{0}\left(2x+3\right)^2\,dx$. **4**

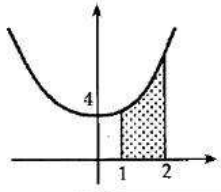
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
.	4	2.2	4						2.2.4	2.2.5	Source: 1996 P1 qu.5

•1	$\frac{1}{3}(2x+3)^3$	OR	•1 $\frac{4}{3}x^3$
•2	$+2$		•2 $6x^2+9x$
•3	$\frac{1}{6}(3)^3-\frac{1}{6}(-6+3)^3$		•3 $[0]-\left[\frac{4}{3}(-3)^3+6(-3)^2+9(-3)\right]$
•4	9		•4 9

[SQA] 14. Evaluate $\int_{1}^{2}\left(3x^2+4\right)\,dx$ and draw a sketch to illustrate the area represented by this integral. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.2
.	5	2.2	5						2.2.5	2.2.6	Source: 1990 P1 qu.6

•1 x^3
•2 $4x$
•3 11
•4 sketch of parabola with min above origin
•5 shade from 1 to 2



[SQA] 15.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(a)	2	0.1	2						0.1		Source
(b)	3	2.2	3						2.2.7		1990 P1 qu.13

- •1 $2x^2 = 4x - 2x^2$ *or* $y = 4 - y$
- •2 $x = 1$ *and* $x = -1$
- •3 $\int_{-1}^{1} \left(4 - 2x^2 - 2x^2\right) dx$
- •4 $4x - \frac{2}{3}x^3 - \frac{2}{3}x^3$
- •5 $5\frac{1}{3}$

[SQA] 16. For all points on the curve $y = f(x)$, $f'(x) = 1 - 2x$.

If the curve passes through the point $(2, 1)$, find the equation of the curve. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	4	2.2	4						2.2.8		Source
											1990 P1 qu.8

- •1 $\int (1 - 2x)dx = \ldots\ldots$
- •2 $x - x^2$
- •3 $+c$
- •4 $c = 3$

[SQA] 17. A curve for which $\frac{dy}{dx} = 3x^2 + 1$ passes through the point $(-1, 2)$.

Express y in terms of x. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	4	2.2	4						2.2.8		Source
											1992 P1 qu.4

- •1 $\int \left(3x^2 + 1\right) dx$
- •2 $x^3 + x$
- •3 $+c$
- •4 $y = x^3 + x + 4$

[SQA] 18. A curve for which $\frac{dy}{dx} = 6x^2 - 2x$ passes through the point $(-1, 2)$.

Express y in terms of x. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	3	2.2	3						2.2.8		Source: 1998 P1 qu.10

- •1 $y = 2x^3 - x^2$
- •2 $y = 2x^3 - x^2 + k$ and substituting
- •3 $k = 5$

[SQA] 19. A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	C	NC	C18, C22	$V = -\frac{4}{3}(4-t)^{\frac{3}{2}} + \frac{32}{3}$	2002 P2 Q8

- •1 ss: know to integrate acceleration
- •2 pd: integrate
- •3 ic: use initial conditions with const. of int.
- •4 pd: process solution

- •1 $V = \int (2(4-t)^{\frac{1}{2}})\,dt$ *stated or implied by* •2
- •2 $2 \times \frac{1}{-\frac{3}{2}}(4-t)^{\frac{3}{2}}$
- •3 $0 = 2 \times \frac{1}{-\frac{3}{2}}(4-0)^{\frac{3}{2}} + c$
- •4 $c = 10\frac{2}{3}$

[SQA] 20. A curve with equation $y = f(x)$ passes through the point $(2, -1)$ and is such that $f'(x) = 4x^3 - 1$.

Express $f(x)$ in terms of x. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	5	2.2	5						2.2.8		Source: 1991 P1 qu.10

- •1 $\int (4x^3 - 1)\,dx = \ldots\ldots$
- •2 $x^4 - x$
- •3 $+c$
- •4 $f(2) = 14 + c$
- •5 $c = -15$

[SQA] 21. The graph of $y = g(x)$ passes through the point $(1, 2)$.

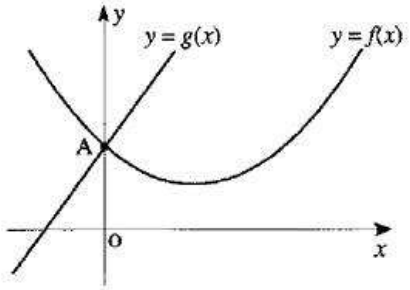
If $\frac{dy}{dx} = x^3 + \frac{1}{x^2} - \frac{1}{4}$, express y in terms of x. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	4	2.2	4						2.2.8		Source: **1999 P1 qu.11**

- $\bullet^1$ x^{-2} *stated or implied by* $\bullet^2$ *or* $\bullet^3$
- $\bullet^2$ $y = \int (x^3 + x^{-2} - \frac{1}{4})dx$ **or** the appearance of any term of $\frac{1}{4}x^4 - \frac{1}{4}x - x^{-1}$
- $\bullet^3$ the remaining two terms
- $\bullet^4$ $c = 3$

[SQA] 22. The graphs of $y = f(x)$ and $y = g(x)$ intersect at the point A on the y-axis, as shown in the diagram.

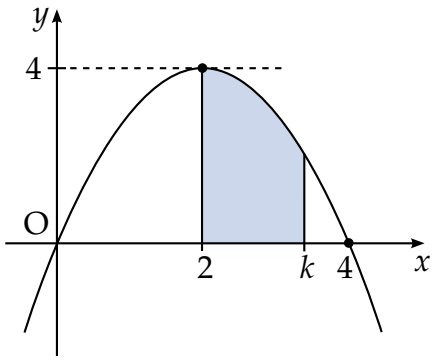
If $g(x) = 3x + 4$
and $f'(x) = 2x - 3$, find $f(x)$. **4**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
.	4	2.2	4						2.2.4	1.2.9	Source: **1993 P1 qu.11**

- $\bullet^1$ $\int f'(x)dx$
- $\bullet^2$ $x^2 - 3x$
- $\bullet^3$ use (0, 4) to find c
- $\bullet^4$ $f(x) = x^2 - 3x + 4$

[SQA] 23. The parabola shown crosses the x-axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.



The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.

(*a*) Find the equation of the parabola. **2**

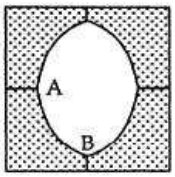
(*b*) Hence show that the shaded area, A, is given by

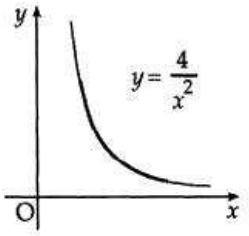
$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. **3**

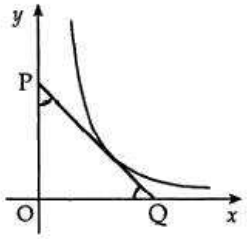
Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(*b*)	3	C	CN	C16	proof	

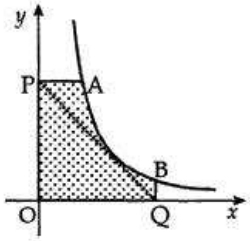
•1 ic: state standard form •2 pd: process for x^2 coeff. •3 ss: know to integrate •4 pd: integrate correctly •5 pd: process limits and complete proof	•1 $ax(x-4)$ •2 $a = -1$ •3 $\int_2^k$(function from (a)) •4 $-\frac{1}{3}x^3 + 2x^2$ •5 $-\frac{1}{3}k^3 + 2k^2 - \left(-\frac{8}{3} + 8\right)$

[SQA] 24. The makers of "OLO", the square mint with the not-so-round hole, commissioned an advertising agency to prepare an illustration to the specification described in (i) to (iii) below. The finished illustration will look like the diagram on the right.









(i) The curve AB in the finished illustration is part of the curve with equation $y = \frac{4}{x^2}$.

(ii) A tangent to this curve, making equal angles with both axes, is to be drawn as shown (line PQ)

(iii) Straight lines perpendicular to the axes are to be drawn from P and Q as shown. The shaded part forms $\frac{1}{4}$ of the finished illustration.

(*a*) State the gradient of PQ and hence find the point of contact of the tangent PQ with the curve. **(5)**

(*b*) Find the equation of PQ and the coordinates of A and B. **(4)**

(*c*) Calculate the shaded area of the finished illustration. **(6)**

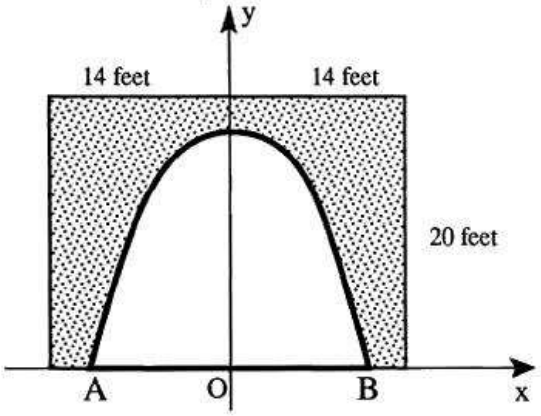
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	5	1.1	3	2					1.1.3,	1.1.10	Source
(*b*)	4	1.1	1	3					1.1.7,	0.1	**1989 Paper 2**
(*c*)	6	2.2		6					2.2.6		**Qu. 10**

(*a*)
- •1 $m_{PQ} = -1$
- •2 $f(x) = 4x^{-2}$
- •3 $f'(x) = -8x^{-3}$
- •4 $-8x^{-3} = -1$
- •5 $x = 2$ and $f(2) = 1$

(*b*)
- •6 $x + y = 3$
- •7 $\frac{4}{x^2} = 3$
- •8 $x \approx 1.15$
- •9 A(1.15,3), B(3,0.44)

(*c*)
- •10 suitable division of area
- •11 rectangle $OPA'C' = 3 \times 1.15 = 3.45$
- •12 curved area $QBA'C' = \int_{1.15}^{3} \frac{4}{x^2}\,dx$
- •13 $\left[-\frac{4}{x}\right]_{1.15}^{3}$
- •14 2.15
- •15 $(3.45 + 2.15) \times 4 = 22.4$

[SQA] 25. The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot. The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	2	0.1	2						0.1		Source
(*b*)	4	2.2	4						2.2.6		1993 Paper 2 Qu.2

(*a*) •1 $18 - \frac{1}{8}x^2 = 0$

•2 $x = \pm 12$

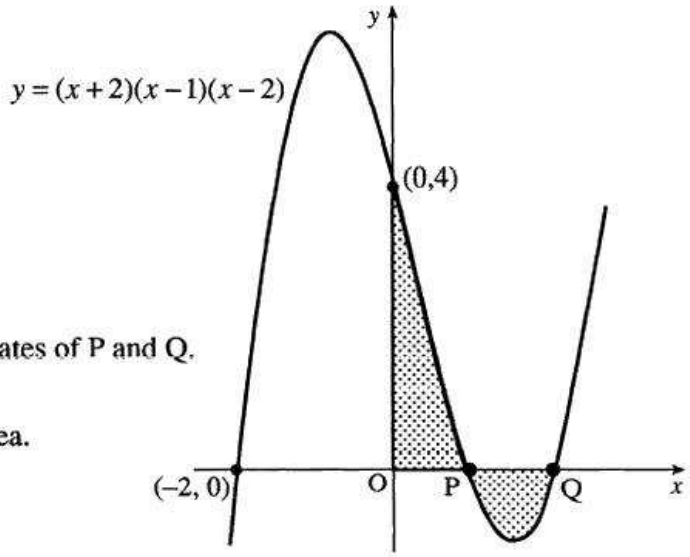
(*b*) •3 $Area = 2\int_0^{12} y\ dx$

•4 integrating

•5 288

•6 for knowing to subtract area of parabola from area of rectangle and multiply by 3.

[SQA] 26. The diagram shows a sketch of the graph of $y = (x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.



(a) Write down the coordinates of P and Q. **(2)**

(b) Find the total shaded area. **(7)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	2.1	2						2.1.2		Source
(b)	7	2.2	6	1					2.2.6		1997 Paper 2 Qu.4

(a) $\bullet^1$ (1,0)

$\bullet^2$ (2,0)

(b) $\bullet^3$ $\int f(x)dx$

$\bullet^4$ $\int_0^1 - \int_1^2$

$\bullet^5$ $(x+2)\left(x^2-3x+2\right)$ or equiv.

$\bullet^6$ $x^3 - x^2 - 4x + 4$

$\bullet^7$ $\frac{1}{4}x^4 - \frac{1}{3}x^3 - 2x^2 + 4x$

$\bullet^8$ $1\frac{11}{12}$ or $-\frac{7}{12}$

$\bullet^9$ $2\frac{1}{2}$

[SQA] 27. A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

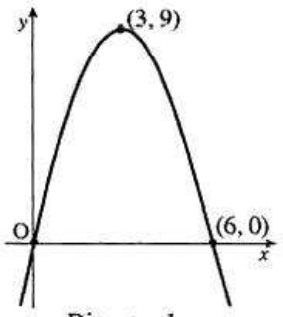


Diagram 1

(a) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(b) Find the area enclosed by the parabola and the x-axis. **(4)**

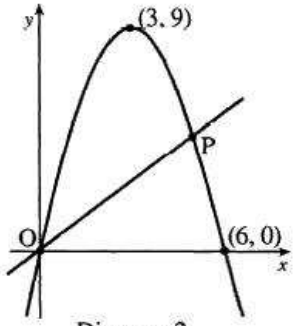


Diagram 2

(c) (i) Diagram 2 shows the parabola from (a) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	2	1.2	2						1.2.7		Source
(b)	4	2.2	4						2.2.6		**1998 Paper 2**
(c)i	2	2.1	2						2.1.8		**Qu. 4**
(c)ii	3	2.2		3					2.2.7		

(a) •1 $a = 1$

•2 $b = 6$

(b) •3 $\int_0^6 x(6-x)\,dx$

•4 $\int \left(6x - x^2\right) dx$

•5 $3x^2 - \frac{1}{3}x^3$

•6 36

(c) •7 $x = 6x - x^2$

•8 $x_P = 5$

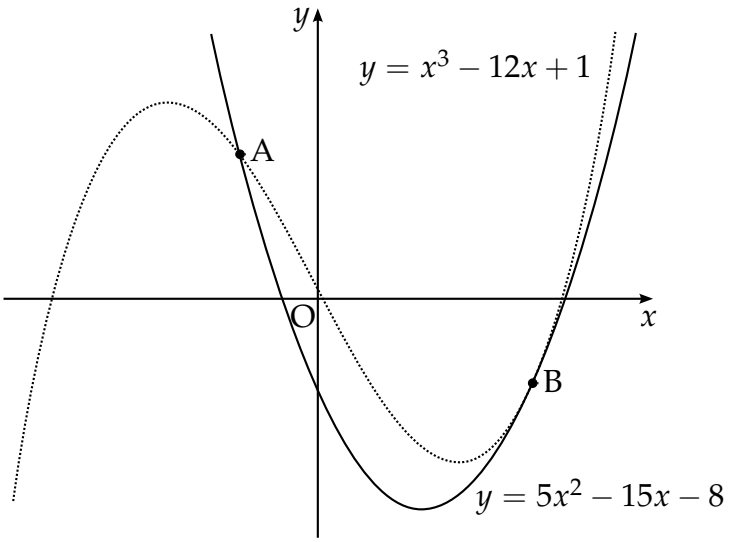
•9 $\int_0^5 \left(6x - x^2 - x\right) dx$ or equiv.

•10 $\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ or equiv.

•11 $\frac{125}{6}$ or equiv.

[SQA] 28. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	0.1	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

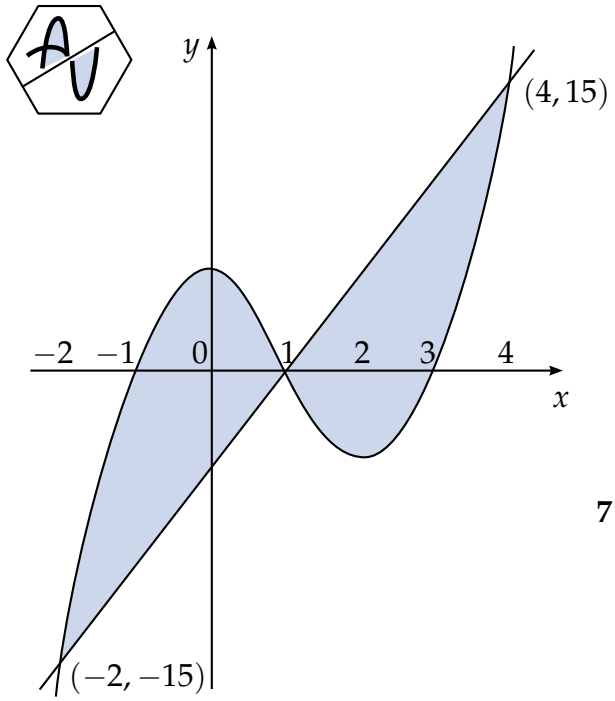
•[1] ss: know to diff. and equate •[2] pd: differentiate •[3] pd: form equation •[4] ic: interpret solution •[5] ic: interpret diagram •[6] ss: know how to find area between curves •[7] ic: interpret limits •[8] pd: form integral •[9] pd: process integration •[10] pd: process limits	•[1] find derivatives and equate •[2] $3x^2 - 12$ **and** $10x - 15$ •[3] $3x^2 - 10x + 3 = 0$ •[4] $x = 3$, $x = \frac{1}{3}$ •[5] tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident •[6] $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$ •[7] $\int_{-1}^{3} \cdots dx$ •[8] $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv. •[9] $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv. •[10] $21\frac{1}{3}$

[SQA] 29. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area. **7**

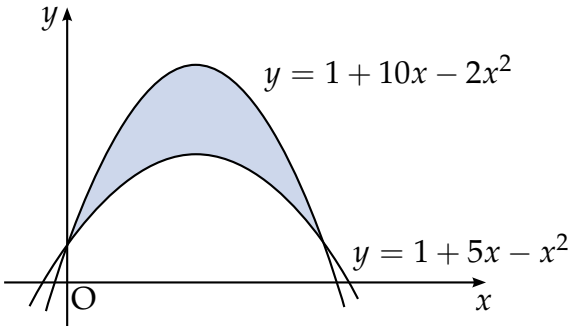


Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	7	C	CN	C17	$40\frac{1}{2}$ units^2	2001 P2 Q8

- $\bullet^1$ ss: express in standard form
- $\bullet^2$ ss: split area and integrate
- $\bullet^3$ ss: subtract functions
- $\bullet^4$ pd: process
- $\bullet^5$ pd: process
- $\bullet^6$ pd: process
- $\bullet^7$ ic: use symmetry or otherwise for total area

- $\bullet^1$ $y = x^3 - 3x^2 - x + 3$
- $\bullet^2$ $\int_1^4(\ldots)dx$ **or** $\int_{-2}^1(\ldots)dx$
- $\bullet^3$ $\int \left[(5x-5) - (x^3 - 3x^2 - x + 3)\right] dx$ **or** $\int \left[(x^3 - 3x^2 - x + 3) - (5x - 5)\right] dx$
- $\bullet^4$ $\int(-x^3 + 3x^2 + 6x - 8)dx$
- $\bullet^5$ $\left[-\frac{1}{4}x^4 + x^3 + 3x^2 - 8x\right]$
- $\bullet^6$ $20\frac{1}{4}$ **or** $-20\frac{1}{4}$ *depending on chosen integrals*
- $\bullet^7$ $40\frac{1}{2}$

[SQA] 30. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
	6	C	CN	C17	$20\frac{5}{6}$	2002 P2 Q5

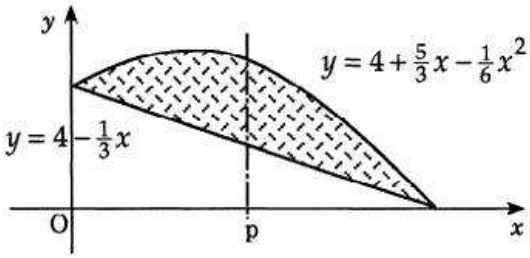
•[1] ss: find intersections •[2] ss: know to find limits •[3] ss: know to integrate (upper − lower) •[4] pd: simplify •[5] pd: integrate •[6] pd: process limits	•[1] $1 + 10x - 2x^2 = 1 + 5x - x^2$ •[2] $x = 0, 5$ **and** $\int_0^5 ()$ •[3] $\int ((1 + 10x - 2x^2) - (1 + 5x - x^2))\, dx$ •[4] $\int (5x - x^2)\, dx$ •[5] $\frac{5}{2}x^2 - \frac{1}{3}x^3$ •[6] $20\frac{5}{6}$

[SQA] 31. When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.

(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**



(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.

By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.

(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	5	2.2	5						2.2.7		Source
(*b*)	3	2.2		3					2.2.5		**1995 Paper 2**
(*c*)	5	2.1	2	3					2.1.3		**Qu.10**

(*a*) •1 Area under curve - area under line

•2 abscissae at intersection are 0 **and** 12

•3 $\int_0^{12} \left(4 + \frac{5}{3}x - \frac{1}{6}x^2 - \left(4 - \frac{1}{3}x\right)\right) dx$

•4 $\left[x^2 - \frac{1}{18}x^3\right]_0^{12}$ or equivalent

•5 48

(*b*) •6 $\int \ldots\ldots dx = 24$

•7 $\int_0^p \left(2x - \frac{1}{6}x^2\right) dx = 24$ or equivalent statement

•8 $p^2 - \frac{1}{18}p^3 = 24$

(*c*) •9 "$f(6) = 0$" or equivalent

•10 divide by $(p - 6)$

•11 $p^2 - 12p - 72$

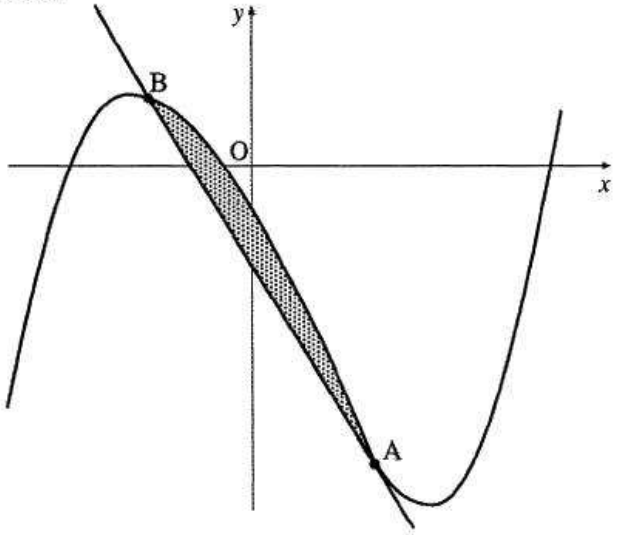
•12 $p = 6 \pm \sqrt{108}$ or equivalent

•13 outside range 0 – 12

[SQA] 32. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**

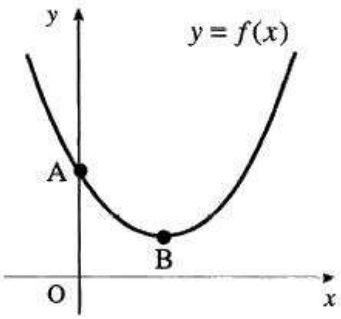


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.2
(*a*)	8	2.1					5	3	2.1.8, 1.1.7, 1.3.9	Source **1996 Paper 2 Qu.8**
(*b*)	3	2.2						3	2.2.7	

(*a*)
- •1 *strat*: $\frac{dy}{dx} = \ldots$
- •2 $\frac{dy}{dx} = 3x^2 - 2x - 6$
- •3 $m_{tgt} = -5$
- •4 $y + 8 = -5(x - 1)$
- •5 *strat*: attempt to simplify and equate y's
- •6 $x^3 - x^2 - x + 1 = 0$
- •7 *strat*: *e.g.* try to factorise
- •8 $B = (-1, 2)$

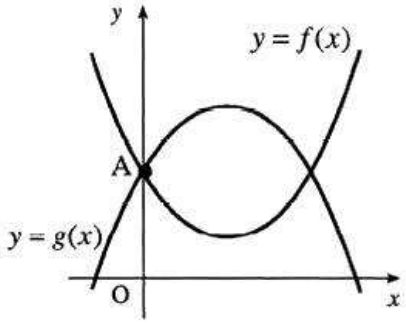
(*b*)
- •9 $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$
- •10 $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$
- •11 $1\frac{1}{3}$

[SQA] 33. The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$. The graph cuts the y-axis at A and has a minimum turning point at B.



(*a*) Write down the coordinates of A and B. **(3)**

(*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n. **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.2
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	3	1.2	3						1.2.9		Source
(*b*)	5	2.2	5						2.2.7		1997 Paper 2
(*c*)	3	0.1		2					0.1		Qu.5

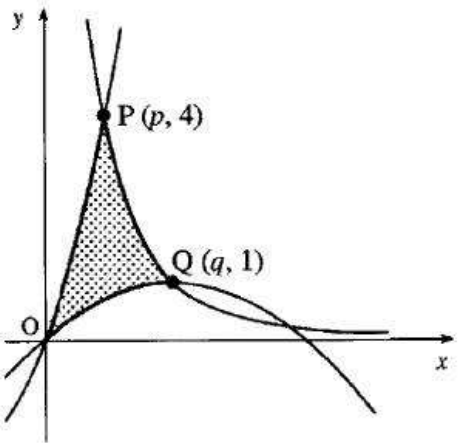
(*a*)
- •1 $A = (0,5)$
- •2 $x_B = 2$
- •3 $y_B = 1$

(*b*)
- •4 $\int_0^4$
- •5 $\int \left(\left(5 + 4x - x^2\right) - \left(x^2 - 4x + 5\right) \right) dx$
- •6 $8x - 2x^2$ *or equiv.*
- •7 $4x^2 - \frac{2}{3}x^3$ *or equiv.*
- •8 $\frac{64}{3}$

(*c*)
- •9 $n = -1$
- •10 $m = 10$

[SQA] 34. The origin, O, and the points P and Q are the vertices of a curved 'triangle' which is shaded in the diagram. The sides lie on curves with equations

$y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.



(a) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. 2

(b) Calculate the shaded area. 7

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(a)	2	1.2	2						1.2.9		Source
(b)	7	2.2		7					2.2.7		**1999 Paper 2 Qu. 10**

(a) •1 $p = 1$

•2 $q = 2$

OR

(b) •3 $\int_0^1 (\text{'}OP\text{'} - \text{'}OQ\text{'})\,dx + \int_1^2 (\text{'}PQ\text{'} - \text{'}OQ\text{'})\,dx$

•4 $\int_0^1 \left(x^2 + 3x - x + \frac{1}{4}x^2\right) dx$

•5 $\left[\frac{5}{12}x^3 + x^2\right]$ *or* $\left[\frac{1}{3}x^3 + \frac{3}{2}x^2 - \frac{1}{2}x^2 + \frac{1}{12}x^3\right]$

•6 $\frac{17}{12}$

•7 $\int_1^2 \left(4x^{-2} - x + \frac{1}{4}x^2\right) dx$

•8 $\left[-4x^{-1} - \frac{1}{2}x^2 + \frac{1}{12}x^3\right]$

•9 $\frac{13}{12}$ and Area $= 2\frac{1}{2}$

•3 $\int_0^1 \ldots\ldots\,dx + \int_1^2 \ldots\ldots\,dx - \int_0^2 \ldots\ldots\,dx$

•4 $\int_0^1 \left(x^2 + 3x\right) dx + \int_1^2 \left(4x^{-2}\right) dx - \int_0^2 \left(x - \frac{1}{4}x^2\right) dx$

•5 $\left[\frac{1}{3}x^3 + \frac{3}{2}x^2\right]$

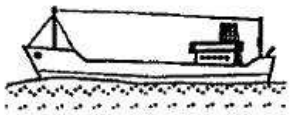
•6 $\left[-4x^{-1}\right]$

•7 $\left[\frac{1}{2}x^2 - \frac{1}{12}x^3\right]$

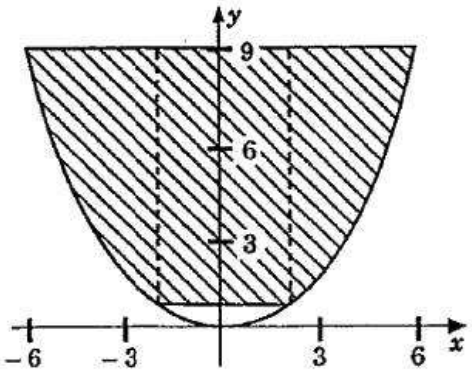
•8 any two evaluations from $\frac{11}{6}, 2, \frac{4}{3}$

•9 third evaluation and area $= \frac{11}{6} + 2 - \frac{4}{3} = 2\frac{1}{2}$

[SQA] 35. **The cargo space of a small bulk carrier is 60m long.**



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \le x \le 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.



(9)

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.2 Source 1994 Paper 2 Qu.10
-	9	2.2					3	6	2.2.7, 0.1	

(-) •1 strategy: split into approp. parts

•2 $y = 1 \Rightarrow x = \pm 2$

•3 first rectangular area

•4 $9 - \frac{1}{4}x^2$ for integrand of shaped area

•5 $\int_2^5 dx$ for limits of shaped area

•6 for integrating.....$\left(9x - \frac{1}{12}x^3\right)$

•7 for evaluating.....$\left(\frac{56}{3}\right)$

•8 total cross-sectional area $= \frac{208}{3}(m^2)$

•9 volume $= 4160\ (m^3)$

[SQA] 36. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

(*a*) Find formulae for

(i) $f\big(g(x)\big)$

(ii) $g\big(f(x)\big)$. **4**

(*b*) The function h is defined by $h(x) = f\big(g(x)\big) + g\big(f(x)\big)$.

Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. **3**

(*c*) Find the area enclosed between this graph and the x-axis. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	4	1.2	4						1.2.6		Source 1999 Paper 2 Qu. 6
(*b*)	3	1.2	3						1.2.9	0.1	
(*c*)	4	2.2	4						2.2.6		

(*a*) •1 $f\left(x^2\right)$ *stated or implied by* •2

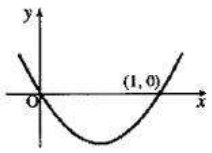
•2 $x^2 - 1$

•3 $g(x-1)$ *stated or implied by* •4

•4 $(x-1)^2$

(*b*) •5 $(x-1)^2 + x^2 - 1$ and complete proof

•6 sketch as shown



•7 minimum at $\left(\frac{1}{2}, -\frac{1}{2}\right)$ calculated or on sketch

(*c*) •8 $\int_0^1 \left(2x^2 - 2x\right) dx$

•9 $\left[\frac{2}{3}x^3 - x^2\right]$

•10 $-\frac{1}{3}$

•11 dealing with – ve

[SQA] 37. A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbb{R}$.

(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **6**

(*c*) Sketch the curve $y = f(x)$. **2**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.2
(*a*)	2	1.2	2						1.2.9		Source
(*b*)	6	1.3	6						1.3.12		1989 Paper 2 Qu. 1
(*c*)	2	1.3	2						1.3.13		
(*d*)	4	2.2	4						2.2.6		

(*a*) •1 (0,0)
•2 (3,0)

(*b*) •3 $f'(x) = 12x^2 - 24x$
•4 $f'(x) = 0$ **stated explicitly**
•5 $x = 0,\ x = 2$
•6

x	0^-	0	0^+	2^-	2	2^+
f'	+	0	−	−	0	+

•7 max at (0,0)
•8 min at (2,−16)

(*c*) •9 correct shape
•10 (0,0),(3,0),(2,−16) annotated

(*d*) •11 $\int_0^3 \left(4x^3 - 12x^2\right) dx$
•12 area $= -\int_0^3 \left(4x^3 - 12x^2\right) dx$
•13 $\left[-x^4 + 4x^3\right]_0^3$
•14 27

[END OF WRITTEN QUESTIONS]