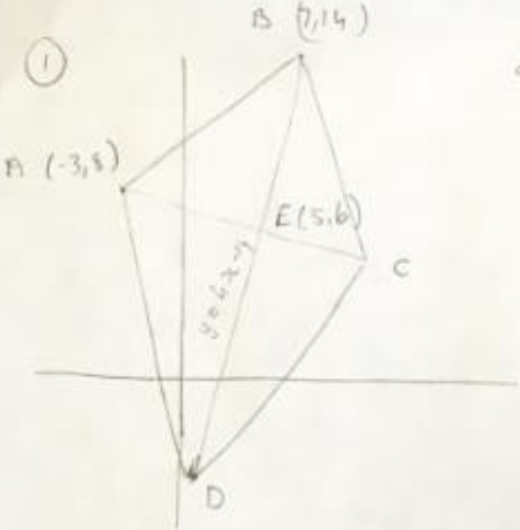
1



a) $y = 4x - 14$

$m = 4$

$\therefore m_{AC} = -\frac{1}{4}$

Eqn AC $(-3,8)$ $m = -\frac{1}{4}$

$y - 8 = -\frac{1}{4}(x+3)$

$4y - 32 = -x - 3$

$4y = -x + 29$

$x + 4y - 29 = 0$

$a = 1 \quad b = 4 \quad c = -29$

b) $y = 4x - 14 \quad \overset{x-4}{\Rightarrow} \quad -4y = -16x + 56$

$4y = -x + 29 \qquad\qquad \underline{4y = -x + 29}$

$0 = -17x + 85$

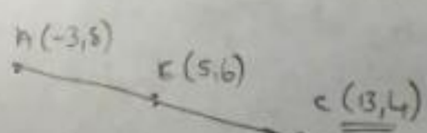
$x = 5$

when $x = 5$, $y = 20 - 14$

$= 6$

$E(5,6)$

c)



2 $f(x) = 4x+1 \quad g(x) = \frac{1}{x-1}$

a) $f(g(x))$

$= f\left(\frac{1}{x-1}\right)$

$= 4\left(\frac{1}{x-1}\right) + 1$

$= \frac{4}{x-1} + \frac{x-1}{x-1}$

$h(x) = \frac{x+3}{x-1}$ as req

b) $h(\sqrt{5}) = \frac{\sqrt{5}+3}{\sqrt{5}-1}$

$= \frac{\sqrt{5}+3}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{5 + 4\sqrt{5} + 3}{5 - 1}$

$= \frac{8 + 4\sqrt{5}}{4}$

$= 2 + \sqrt{5}$

$p = 2$

(3) $f(x) = 2x^3 + 3x^2 - 12x + 7$

$f'(x) = 6x^2 + 6x - 12$ $f'(1) = 0$

SP when $f'(x) = 0$

$6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$x = -2 \quad x = +1$

when $x = -2$, $f(x) = -16 + 12 + 24 + 7$

$= 27$

$(-2, 27)$ Q

when $x = 1$, $f(1) = 2 + 3 - 12 + 7$

$= 0$

$(1, 0)$ P

b) $\int_{-2}^{1} (2x^3 + 3x^2 - 12x + 7)\,dx$

$= \left[\frac{2x^4}{4} + \frac{3x^3}{3} - \frac{12x^2}{2} + 7x\right]_{-2}^{1}$

$= \left[\frac{x^4}{2} + x^3 - 6x^2 + 7x\right]_{-2}^{1}$

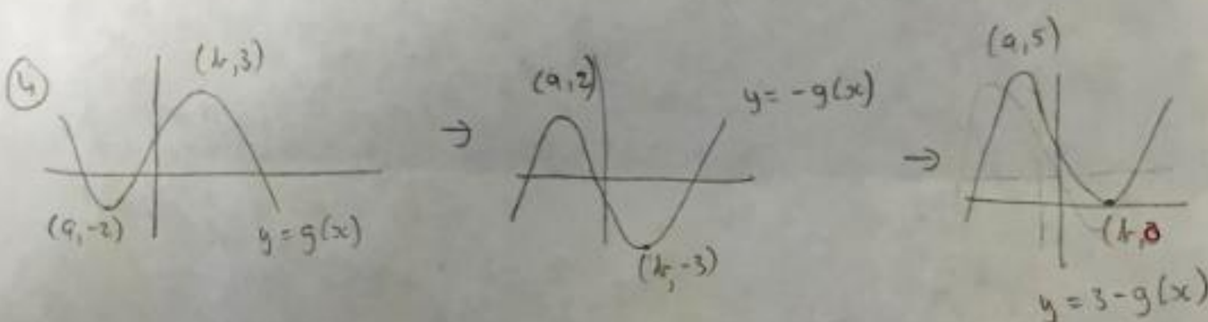
$= \left(\frac{1}{2} + 1 - 6 + 7\right) - (8 - 8 - 24 - 14)$

$= \left(2\frac{1}{2}\right) - (-38)$

$= 40\frac{1}{2}$

Area $= 40\frac{1}{2}$ sq units

(4)



(5) $U_{n+1} = \frac{a}{4} U_n + 12$

a) $U_0 = 16$

$U_1 = \frac{a}{4}(16) + 12$

$= 4a + 12$

$U_2 = \frac{a}{4}(4a + 12) + 12$

$= a^2 + 3a + 12$

as req

b) $a^2 + 3a + 12 = 30$

$a^2 + 3a - 18 = 0$

$(a+6)(a-3) = 0$

$a = -6$ or $a = 3$

But $a > 0 \therefore a = 3$

c) $U_{n+1} = \frac{3}{4} U_n + 12$

Limit because $-1 < \frac{3}{4} < 1$

$L = \frac{12}{\frac{1}{4}}$

$= 48$

⑥

$$\begin{array}{r|cccc} 2 & 2 & p & q & 4 \\ & 0 & 4 & 8+2p & 16+4p+2q \\ \hline & 2 & 4+p & 8+2p+q & \boxed{20+4p+2q} \end{array} = 0$$

$$\begin{array}{r|cccc} -1 & 2 & p & q & 4 \\ & 0 & -2 & -p+2 & p-2-q \\ \hline & 2 & p-2 & -p+2+q & \boxed{p+2-q} \end{array} = 9 \qquad p-q=7$$

$20+4p+2q = 0 \Rightarrow 4p+2q = -20$

$p-q = 7 \Rightarrow 2p-2q = 14$

$6p = -6$

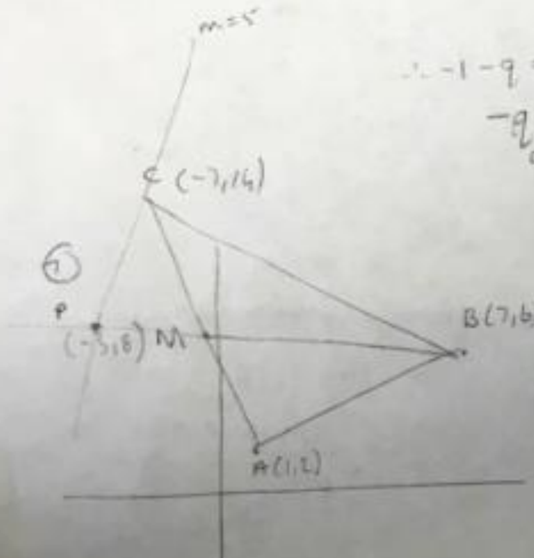
$p = -1$

$\therefore -1-q = 7$

$-q = 8$

$q = -8$

⑦



a) $m_{AB} = \frac{6-2}{7-1} = \frac{4}{6} = \frac{2}{3}$ $\qquad m_{AC} = \frac{14-2}{-7-1} = \frac{12}{-8} = -\frac{3}{2}$

$m_{AB} \times m_{AC} = -1$

$\therefore AB \perp AC$

b) $M\left(\frac{-7+1}{2}, \frac{14+2}{2}\right)$

$= (-3, 8)$

$m_{BM} = \frac{6-8}{7+3} = \frac{-2}{10} = \frac{-1}{5}$

m_{BM} $(7,6)$ $m = -\frac{1}{5}$

$y-6 = -\frac{1}{5}(x-7)$

$5y-30 = -x+7$ $\Rightarrow 5y = -x+37$

c) $C(-7,14)$ $m=5$

$y-14 = 5(x+7)$

$y-14 = 5x+35$

$y = 5x+49$

$5y = -x+37 \Rightarrow 5y = -x+37$

$y = 5x+49 \xRightarrow{\times 5} -5y = -25x-245$

$0 = -26x - 208$

$26x = -208$

$x = -8$

when $x=-8$, $y = 5(-8)+49$

$= 9$

$P(-8, 9)$

(8) $f(x) = x^3 - 4x + 14$

a) $f(p) = p^3 - 4p + 14 = 14$

$p^3 - 4p = 0$

$p(p^2 - 4) = 0$

$p = 0 \quad p = 2 \text{ or } -2$

$p = 2$ because $p > 0$

b) $f'(p) = 3p^2 - 4$

$f'(2) = 12 - 4$

$= 8$

$f'(2)$ is +ve $\therefore f(p)$ is increasing when $p = 2$

(9) $y = x^3 - 3x^2 - 9x - 5$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

SP when $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$(x-3)(x+1) = 0$

$x = 3 \quad x = -1$

when $x = 3$, $y = 27 - 27 - 27 - 5$

$= -32$

$A(3, -32)$

b) $\int_{-1}^{5} (x^3 - 3x^2 - 9x - 5)\,dx$

$= \left[\frac{x^4}{4} - \frac{3x^3}{3} - \frac{9x^2}{2} - 5x\right]_{-1}^{5}$

$= \left[\frac{x^4}{4} - x^3 - \frac{9x^2}{2} - 5x\right]_{-1}^{5}$

$= \left(\frac{625}{4} - 125 - \frac{225}{2} - 25\right) - \left(\frac{1}{4} + 1 - \frac{9}{2} + 5\right)$

$= (-106.25) - (1.75)$

$= -108$ Area $= 108$ sq units

(10) $f'(x) = 3x^2 - 10x^{-2}$

$f(x) = \frac{3x^3}{3} - \frac{10x^{-1}}{-1} + c$

$= x^3 + \frac{10}{x} + c$

Sub $x = 2,\ y = 3$

$3 = 8 + 5 + c$

$3 = 13 + c$

$c = -10$

$\therefore f(x) = x^3 + \frac{10}{x} - 10$

b) $f(1) = 1 + 10 - 10$

$= 1$

(11) $H(m) = 4m - \frac{m^2}{1200}$

a) $H(m) = 4m - \frac{1}{1200} m^2$

$H'(m) = 4 - \frac{1}{600} m$

SP when $H'(m) = 0$

$4 = \frac{m}{600}$

$m = 2400$

2400ml needed

b) $H(2400) = 4(2400) - \frac{2400^2}{1200}$

$9600 - 4800$

$= 4800$

Height 4800 m

(12) a) $f(x) = \frac{1}{3}x^3 - 4x^2 + x$

$f'(x) = x^2 - 8x + 1$

$= (x-4)^2 - 15$

$p = -4 \quad q = -15$

b) $(x-4)^2 - 15$

TP $(4, -15)$

min rate of change is -15

when $x = 4$

(13) $U_{n+1} = kU_n - 6 \qquad U_0 = 0$

a) $U_1 = -6$

$U_2 = -6k - 6 = -8$

$-6k = -2$

$k = \frac{1}{3}$

b) (i) $U_{n+1} = \frac{1}{3}U_n - 6$

Limit because $-1 < \frac{1}{3} < 1$

$L = \dfrac{-6}{1-\frac{1}{3}}$

$= \dfrac{-6}{\frac{2}{3}} \qquad -6 \times \frac{3}{2}$

$= -9$

(14) $U_{n+1} = 0.96U_n \qquad U_0 = 40$

a)

After 2 months $\quad 0.96 \times 40 = 38.4$

4 months $\quad 36.864$

6 months $\quad 35.38944 \qquad 0.96^5 \times 40$

8 months $\quad 33.973\ldots$

10 months $\quad 32.614$

$= 32.6$ gigatonnes (3sf)

b) After 1 year $\quad 32.6 + 3.8$

$= 36.414$ g.t.

$0.96^5 \times 36.414$

After a further 10 months $\quad 29.6917\ldots$

After a further year (ie at end of year 2) $\quad 33.4917\ldots$

After 2 years 10 months $\quad 27.308\ldots$ g tonnes

After 3 years $\quad 31.108\ldots$ gtonnes

ie $U_{n+1} = 0.96^5 U_n + 3.8$

c) $L = \dfrac{b}{1-a} =$ $\qquad$ limit because $-1 < 0.96^5 < 1$

$\dfrac{3.8}{1-0.81537}$ $\qquad$ Upper limit $20.582\ldots$ gigatonnes

$L = 20.5820\ldots$ $\qquad$ Lower limit $20.582 - 3.8 = 16.782$ gigatonnes

(15) $f(x) = x^2 + a \quad g(x) = x+1$

a) $g(-2) = -1$

$f(-1) = 1 + a$

$1 + a = -1$

$a = -2$

b) $f(f(x))$

$= f(x^2 - 2)$

$= (x^2-2)^2 + 2$

$= x^4 - 4x^2 + 4 - 2$

$x^4 - 4x^2 + 2$

solve $x^4 - 4x^2 + 2 = 2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x^2 = 0 \quad$ or $\quad x^2 = 4$

$x = 0 \qquad x = \pm 2$

(16) $f(x) = (x-1)^2$

$f(x-3) = (x-4)^2$

$h(x) = (x-4)^2 x^2$

$= (x^2 - 8x + 16)x^2$

$= x^4 - 8x^3 + 16x^2$

as req

b)

$h'(x) = 4x^3 - 24x^2 + 32x$

SP when

$4x^3 - 24x^2 + 32x = 0$

$4x(x^2 - 6x + 8) = 0$

$4x(x-4)(x-2) = 0$

$x = 0$ or $x = 4$ or $x = 2$

when $x = 2$, $y = 16 - 64 + 64$

$= 16$

A $(2, 16)$

(17) $U_{n+1} = pU_n + 6 \qquad U_{n+1} = p^2 U_n + 9$

$L = \frac{6}{1-p} \qquad L = \frac{9}{1-p^2}$

$\frac{6}{1-p} = \frac{9}{1-p^2}$

$6 - 6p^2 = 9 - 9p$

$0 = 6p^2 - 9p + 3$

$0 = 2p^2 - 3p + 1$

$= (p-1)(2p-1)$

$p = 1$ or $p = \frac{1}{2}$ $\quad$ But for limit $-1 < p < 1 \therefore p = \frac{1}{2}$

b) $U_{n+1} = \frac{1}{2}U_n + 6 \quad U_0 = 100 \qquad U_{n+1} = \frac{1}{4} \times 100 + 9$

$= 34$

$U_1 = 50 + 6$

$= 56$

$\therefore$ difference $= 22$

(18)

$$\begin{array}{c|cccc} 1 & 3 & k & 4 & -13 \\ & 0 & 3 & 3+k & 7+k \\ \hline & 3 & 3+k & 7+k & \boxed{k-6} = 0 \end{array}$$

$k = 6$

$y = 3x^3 + 6x^2 + 4x - 13$

$\frac{dy}{dx} = 9x^2 + 12x + 4$

SP when $9x^2 + 12x + 4 = 0$

$(3x+2)(3x+2) = 0$

$3x \quad 2$

$3x \quad 2$

$x = -\frac{2}{3}$

(19) $y = 4x^2 - x^3$, $y = 3x$ } Find Pt of intersection

$\therefore 4x^2 - x^3 = 3x$

$0 = x^3 - 4x^2 + 3x$

$0 = x(x^2 - 4x + 3)$

$0 = x(x-3)(x-1)$

$x = 0$ or $x = 3$ $x = 1$

when $x = 3$, $y = 3(3)$

$= 9$

A $(3,9)$

ii) $y = 4x^2 - x^3$

$\frac{dy}{dx} = 8x - 3x^2$

when $x = 3$, $\frac{dy}{dx} = 24 - 27$

$= -3$ ie $m = -3$

Eqn tangent $(3,9)$ $m = -3$

$y - 9 = -3(x - 3)$

$y - 9 = -3x + 9$

$y = -3x + 18$

(20) $y = x^3 - kx^2 - 16x + 32$

$$\begin{array}{r|rrrr} 2 & 1 & -k & -16 & 32 \\ & 0 & 2 & 4-2k & -24-4k \\ \hline & 1 & 2-k & -12-2k & \boxed{8-4k=0} \end{array} \quad \cdots$$

$k = +2$

b) $y = x^3 - 2x^2 - 16x + 32$

$y = 35$

$x^3 - 2x^2 - 16x + 32 = 35$

$x^3 - 2x^2 - 16x - 3 = 0$

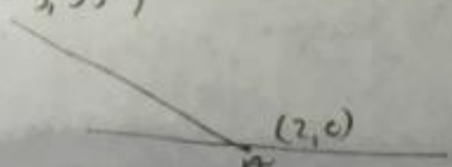
$$\begin{array}{r|rrrr} -3 & 1 & -2 & -16 & -3 \\ & 0 & -3 & 15 & 3 \\ \hline & 1 & -5 & -1 & \boxed{0} \end{array}$$

remainder of zero

$\therefore$ $x = -3$ is a root

i.e. $p = -3$

c) $B(-3, 35)$



$m_{AB} = \dfrac{35-0}{-3-2}$

$= \dfrac{35}{-5} = -7$

$\tan\theta = -7$

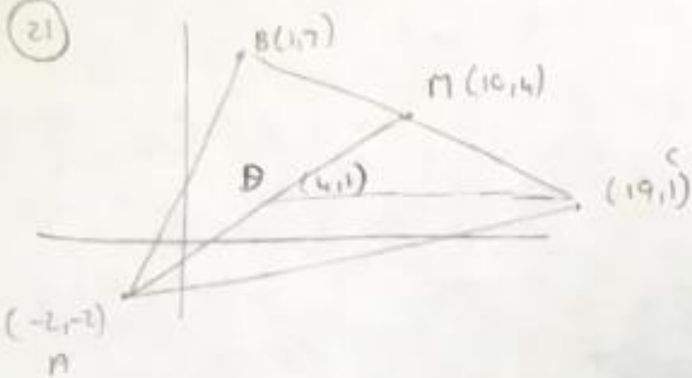
Solve $\tan\theta = 7$

acute $\theta = 81.869\ldots$

req. $\theta = 98.13$

$= 98°$ (nearest $1°$)

21



a) $M = \left(\frac{1+19}{2}, \frac{7+1}{2}\right)$ $\qquad m_{AM} = \frac{4+2}{10+2} = \frac{1}{2}$

$= (10,4)$

Eqn AM $(10,4)$ $m = \frac{1}{2}$

$y - 4 = \frac{1}{2}(x - 10)$

$2y - 8 = x - 10$

$2y = x - 2$

b) $\left.\begin{array}{l} 2y = x - 2 \\ y = 1 \end{array}\right\}$ $\therefore 2 = x - 2$

$x = 4$

D (4,1)

c) $m_{BD} = \frac{7-1}{1-4} = \frac{6}{-3} = -2$

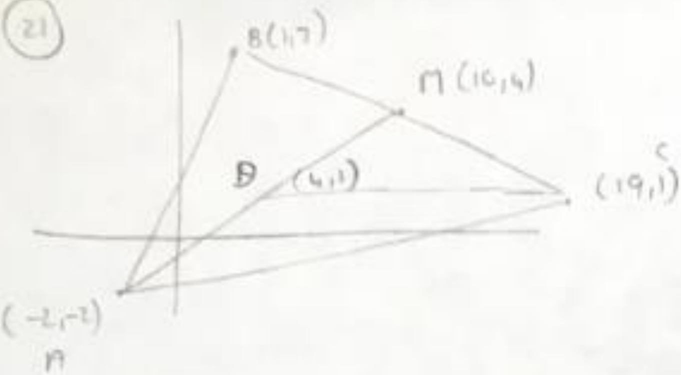
$m_{AM} \times m_{BD}$

$\frac{1}{2} \times -2$

$= -1$

$\therefore AM \perp BD$

21



a) $M = \left(\frac{1+19}{2}, \frac{7+1}{2}\right)$

$= (10,4)$

$m\ AM = \frac{4+2}{10+2} = \frac{1}{2}$

Equ AM $(10,4)$ $m = \frac{1}{2}$

$y-4 = \frac{1}{2}(x-10)$

$2y-8 = x-10$

$2y = x-2$

b) $\left.\begin{array}{l} 2y = x-2 \\ y = 1 \end{array}\right\}$ $\therefore 2 = x-2$

$x = 4$

D $(4,1)$

c) $m\ BD = \frac{7-1}{1-4} = \frac{6}{-3} = -2$

$m\ AM \times m\ BD$

$\frac{1}{2} \times -2$

$= -1$

$\therefore AM \perp BD$

(22) $y = x^3 - \frac{15}{2}x^2 + 12x - 18$

$\frac{dy}{dx} = 3x^2 - 15x + 12$

SP when $3x^2 - 15x + 12 = 0$

$3(x^2 - 5x + 4) = 0$

$(x-4)(x-1) = 0$

$x = 4 \quad x = 1$ (from diag, $x = 1$

when $x = 1$, $y = 1 - \frac{15}{2} + 12 - 18$

$= -12.5$

$(1, -12\frac{1}{2})$ P

ii) crosses x-axis when $y = 0$

$x^3 - \frac{15}{2}x^2 + 12x - 18$

6	1	-7.5	12	-18
	0	6	-9	18
	1	-1.5	3	0

$x = 6$ is a root because remainder of zero

Q $(6, 0)$

(23) $y = x + \frac{p}{\sqrt{x}}$

$x + y = 10$

$y = -x + 10$

$m = -1$

$y = x + px^{-1/2}$

$\frac{dy}{dx} = 1 - \frac{1}{2}px^{-3/2}$

$= 1 - \frac{p}{2\sqrt{x}^3}$

when $x = 4$, $\frac{dy}{dx} = 1 - \frac{p}{2\sqrt{4}^3}$

$= 1 - \frac{p}{16} \leftarrow m$

$1 - \frac{p}{16} = -1$

$2 = \frac{p}{16} \Rightarrow p = \underline{\underline{32}}$

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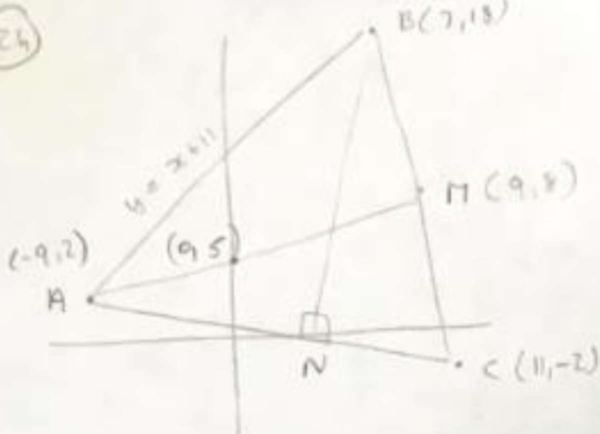

a) $M\left(\frac{7+11}{2}, \frac{18-2}{2}\right)$

$= (9,8)$

$m_{AM} = \frac{8-5}{9-0}$

$= \frac{3}{9}$

$= \frac{1}{3}$

Eqn AM $(0,5)$ $m = \frac{1}{3}$

$y = \frac{1}{3}x + 5$

$3y = x + 15$

b) $\left.\begin{aligned} 3y &= x+15 \\ y &= x+11 \end{aligned}\right\}$

$$\begin{aligned} 3y &= x+15 \\ -\ \ y &= -x-11 \\ \hline 2y &= 4 \\ y &= 2 \end{aligned}$$

When $y=2$, $2 = x+11$

$-9 = x$

$A\ (-9,2)$

c) $m_{AC} = \frac{2+2}{-9-11}$

$= \frac{4}{-20} = -\frac{1}{5}$

$\therefore m_{BN} = 5$

Eqn BN $m=5$ $(7,18)$

$y-18 = 5(x-7)$

$y-18 = 5x-35$

$y = 5x-17$

25 $\int_0^a (2x)\,dx$

$= \left[x^2\right]_0^a$

$= (a^2) - 0$

$= a^2$

$f(a) = 3a^2 - 9a$

$f'(a) = 6a - 9$

$\therefore\ a^2 = 6a - 9$

$a^2 - 6a + 9 = 0$

$(a-3)(a-3) = 0$

$\underline{\underline{a = 3}}$

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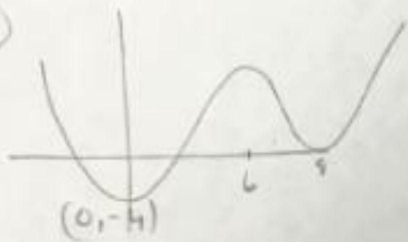
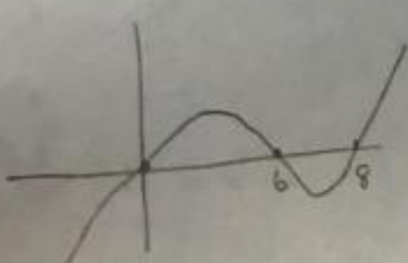


x	$\rightarrow$	0	$\rightarrow$	6	$\rightarrow$	8	$\rightarrow$
$\frac{dy}{dx}$	$-$	0	$+$	0	$-$	0	$+$
In rel to x axis	bel	on	ab	on	bel	on	ab



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$$\begin{array}{r|rrrr} -1 & 2 & 3 & k & -6 \\ & 0 & -2 & -1 & -k+1 \\ \hline & 2 & 1 & k-1 & \boxed{-k-5} = 0 \end{array}$$

$k = -5$

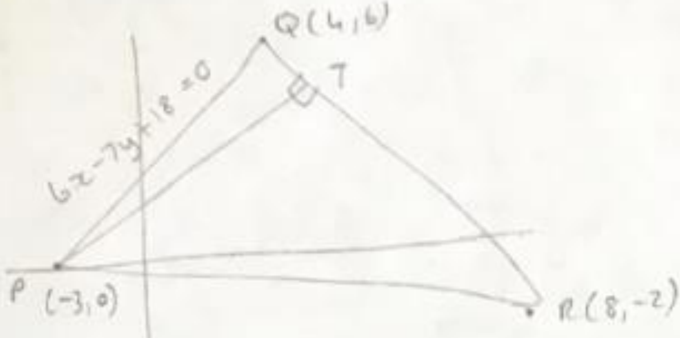
b) $(x+1)(2x^2 + x - 6) = 0$

$(x+1)(2x-3)(x+2) = 0$

$x = -1 \quad x = \frac{3}{2} \quad x = -2$

$$\begin{array}{cc} 2x & -3 \\ x & 2 \end{array}$$

(28)



a) $6x - 7y + 18 = 0$

crosses x-axis when $y = 0$

$$6x - 0 + 18 = 0$$
$$x = -3$$

$P(-3,0)$

b) $m_{QR} = \frac{6+2}{4-8}$

$= \frac{8}{-4}$

$= -2$

$\therefore m_{PT} = \frac{1}{2}$

Eqn PT $(-3,0)$ $m = \frac{1}{2}$

$$y - 0 = \tfrac{1}{2}(x+3)$$
$$2y = x + 3$$

c) Eqn QR $(4,6)$ $m = -2$

$$y - 6 = -2(x-4)$$
$$y - 6 = -2x + 8$$
$$y = -2x + 14$$

Pt of intersection

$$\left.\begin{array}{l} y = -2x + 14 \\ 2y = x + 3 \end{array}\right\} \quad \begin{array}{r} -2y = 4x - 28 \\ 2y = x + 3 \\ \hline 0 = 5x - 25 \end{array}$$

$x = 5$

when $x = 5$, $y = -10 + 14$
$= 4$

$T(5,4)$

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