Firrhill High School

National 5 Mathematics Relationships Unit Assessment Revision Booklet

FORMULAE LIST

The roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Area of a triangle: $A = \frac{1}{2} ab \sin C$

Volume of a sphere: $V = \frac{4}{3} \pi r^3$

Volume of a cone: $V = \frac{1}{3} \pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3} Ah$

Standard deviation: $s = \sqrt{\dfrac{\Sigma(x - \bar{x})^2}{n-1}} = \sqrt{\dfrac{\Sigma x^2 - (\Sigma x)^2 / n}{n-1}}$, where n is the sample size.

1. Find the equations of the straight lines below. Simplify each equation.

   (a)   Gradient = 2   passing through (4, 5)
   (b)   Gradient = 3   passing through (-1, 2)
   (c)   Gradient = -3   passing through (3, -5)
   (d)   Gradient = -1   passing through (-2, -4)
   (e)   Gradient = 4   passing through (2, -1)
   (f)   Gradient = -4   passing through (-3, -2)

2. Solve the inequations.

   (a) $5x + 3 < 3x + 11$ (b) $4y - 4 \geq y + 5$
   (c) $3w + 5 > w - 3$ (d) $8k - 1 \leq 3k - 11$

   (e) $7x - 2 > 3x + 10$ (f) $6x + 10 \geq x - 5$

   (g) $6x + 2 < 3x + 26$ (h) $6m + 2 \leq 4m + 10$

3. (a)   Two fish suppers and three sausage suppers cost £13.
         Write an equation to represent this information.

   (b)   Three adult tickets and four child tickets for a football match cost £75.
         Write an equation to represent this information.

   (c)   Mr and Mrs Dean take their three children to a theme park.
         The total cost of their tickets is £88.
         Write an equation to represent this information.

4. Solve each of these systems of equations algebraically.
   (a) $4x + 3y = 17$ (b) $6m - 4n = 10$ (c) $3a + 5b = 29$
         $5x - y = 7$ $5m + n = 4$ $4a + 3b = 24$

5. Mrs Benson bought 5 fish suppers and two pie suppers for £10·20.
    Mr Brown bought three fish suppers and seven pie suppers for £12·50.
    How much did it cost Mrs Forrest for four fish suppers and three pie suppers?

6. Brian, Molly and their four children visit Waterworld.
    The total cost of their tickets is £56.

    Sarah and her three children visit Waterworld.
    The total cost of their tickets is £36.

    How much will it cost Peter and his two children to visit Waterworld?

7. Change the subject of each formula to x.

    (a) $y = 2x + p$

    (b) $p = u + vx$

    (c) $n = 2mx$

    (d) $y = \dfrac{7x}{8} - 2$

    (e) $a = \dfrac{6x}{5} + 10$

    (f) $M = 3(2x + 1)$

    (g) $h = 4(5x - 3)$

    (h) $d = \dfrac{5x}{2} - 3$

    (i) $c = 4ax + 5$

1. The diagrams below show parabolas with equation $y = kx^2$.
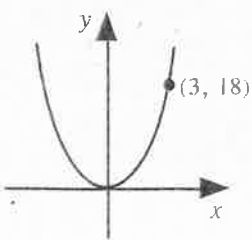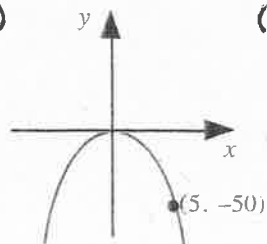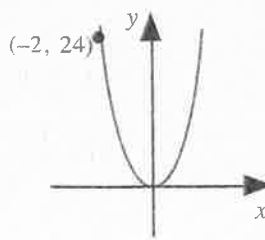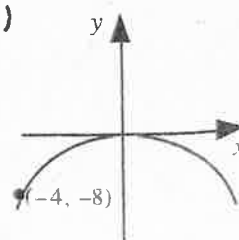   What is the value of k?

a)

b)

(c)
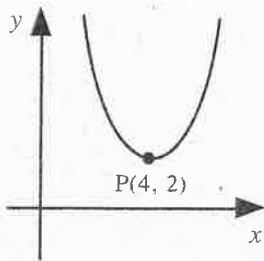

(3, 18)

(d)

(5, -50)

(e)

(-2, 24)

(f)

(-4, -8)
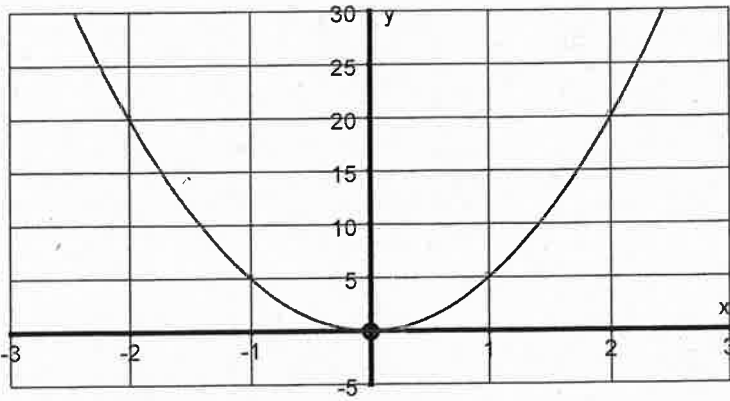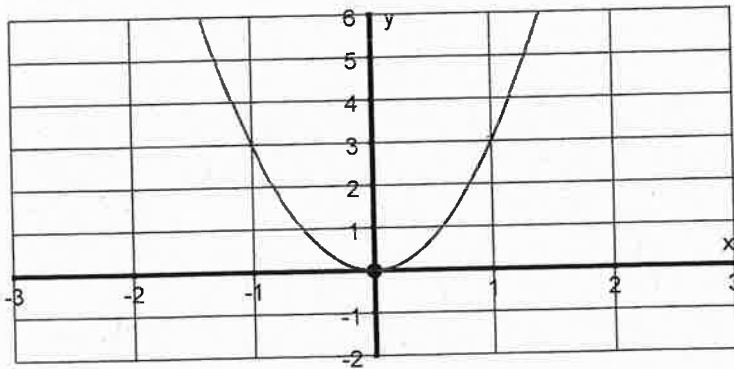
2. Each of these parabolas are of the form $y = (x - a)^2 + b$.
   Write down the values of a and b.

(a)

P(4, 2)

(b)

R(−3, −2)

3.   (a)

B

0   x

A(3, − 9).
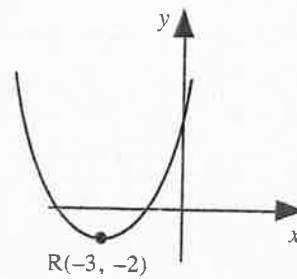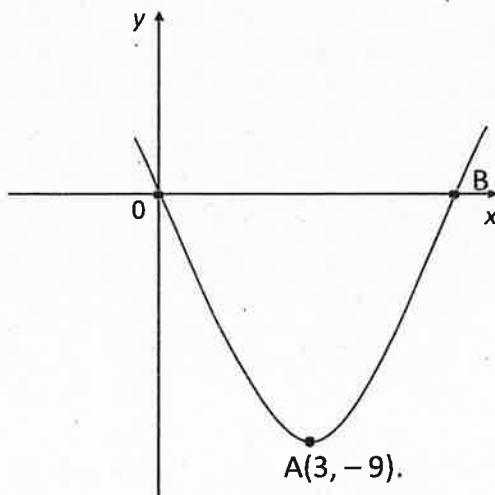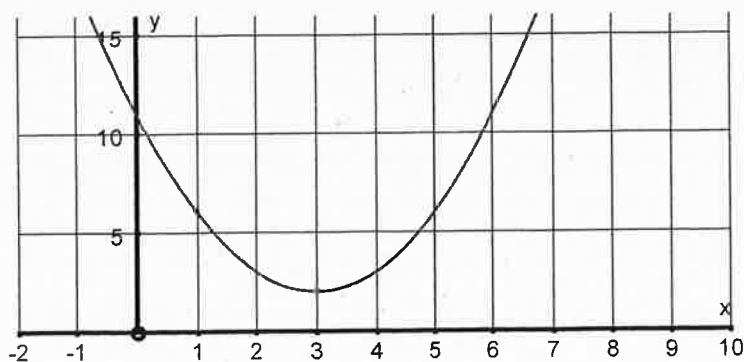
Write down the equation of the parabola in the form $y = (x-a)^2 + b$
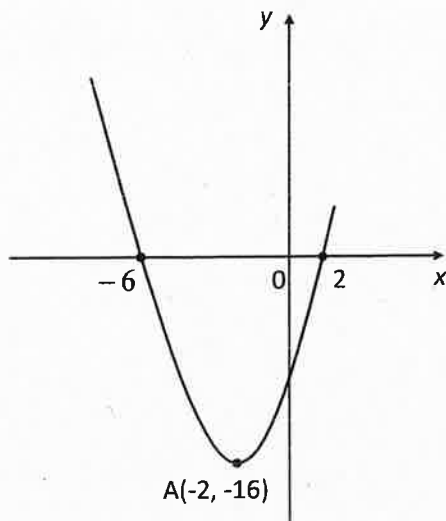
(b)   The graph of a quadratic function is shown below.

The coordinates of its turning point is (3, 2).

The equation of the function can be written in the form $y = (x+a)^2 + b$, where a and b are integers. Write down the values of a and b.

(c)

The equation of the quadratic function whose graph is shown below is of the form $y = (x+a)^2 - b$, where a and b are integers.

Write down the values of a and b.

A(-2, -16)

4. For each parabola below, state

    (i)   the equation of the axis of symmetry.

    (ii)   the coordinates of the turning point.

    (iii)   whether the turning point is a maximum or a minimum.

    (a) $y = (x - 4)^2 - 3$

    (b) $y = (x + 2)^2 + 5$

    (c) $y = (x - 2)^2 + 3$

    (d) $y = (x - 1)^2 + 3$

    (e) $y = (x + 2)^2 - 6$

    (f) $y = (x + 3)^2 + 1$

    (g) $y = (x + 5)^2 - 8$

    (h) $y = (x - 6)^2 + 3$

5. Sketch the graph of each of these parabolas on plain paper.
   Your sketch must clearly show where the graph crosses the x and y axes.
   State the coordinates of the turning point and show this on your graph.

   (a) $y = (x - 4)(x - 6)$
   (b) $y = (x - 6)(x + 4)$
   (c) $y = (x + 8)(x - 4)$
   (d) $y = (x + 2)(x + 6)$
   (e) $y = (x - 8)(x + 4)$
   (f) $y = (x + 3)(x - 5)$

1. Solve the equations.

    (a) $(x - 5)(x - 7) = 0$
    (b) $(x - 3)(x + 2) = 0$
    (c) $(x + 1)(x - 5) = 0$
    (d) $(x + 4)(x + 6) = 0$
    (e) $(2x - 4)(x + 6) = 0$
    (f) $(x + 6)(2x + 6) = 0$
    (g) $(3x + 6)(x - 7) = 0$
    (h) $(x - 6)(2x - 2) = 0$

2. Solve the equations using the quadratic formula.

    (a) $x^2 + 7x + 3 = 0$
    (b) $x^2 - x - 5 = 0$
    (c) $x^2 - 10x - 5 = 0$
    (d) $2x^2 + 12x + 9 = 0$

3. A rocket is fired upwards from the top of a 2m high wall.
    The height of the rocket is given by the formula $h = -4t^2 + 10t + 2$,
    where h is the height above the ground in metres at time t seconds.
    How long will it take the rocket to hit the ground?

4. A missile is fired upwards from the top of a 15m high tower.
    The height of the missile is given by the formula $h = -2t^2 + 20t + 15$,
    where h is the height above the ground in metres at time t seconds.
    How long will it take the missile to hit the ground?

5. A ball is thrown upwards from the top of a 7m high building.
    The height of the ball is given by the formula $h = -3t^2 + 18t + 7$,
    where h is the height above the ground in metres at time t seconds.
    How long will it take the ball to hit the ground?

6.   Calculate the discriminant and determine the number and nature of the roots of the equations.

(a) $x^2 + 8x + 16 = 0$
(b) $x^2 - 3x + 4 = 0$
(c) $x^2 - 6x + 9 = 0$
(d) $x^2 + 8x - 3 = 0$
(e) $x^2 + 4x - 5 = 0$
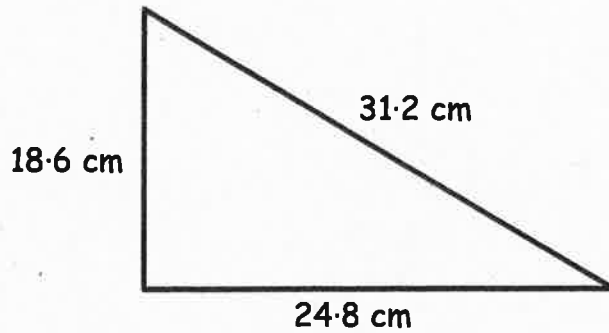(f) $x^2 + 3x + 7 = 0$
(g) $3x^2 - 7x + 2 = 0$
(h) $2x^2 + 3x - 4 = 0$
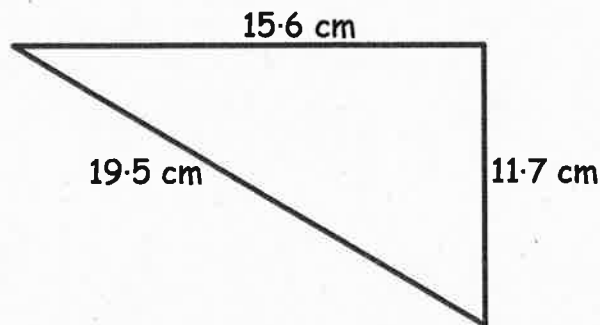(i) $2x^2 - 4x + 2 = 0$
(j) $3x^2 - 5x + 3 = 0$

1. Is this a right angled triangle?

31·2 cm

18·6 cm

24·8 cm

2. A joiner needs to make a right angled triangular shelf to fit into the corner of a room.
   He makes a triangle with dimensions as shown.
   Has he been successful?

15·6 cm

19·5 cm   11·7 cm

3. The perimeter of a traffic island is 10·4m.
   I know one side is 4·3m and another is 4·5m.
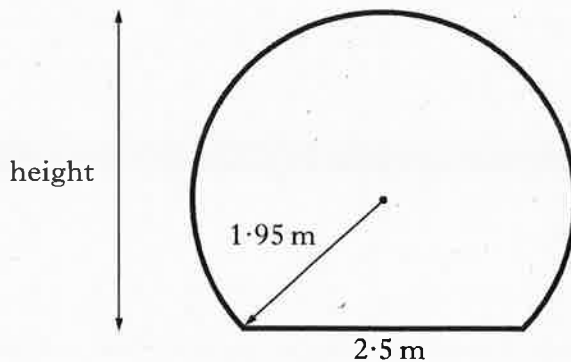   Is the island in the shape of a right angled triangle?
   Explain your answer.

4·3 m   4·5 m
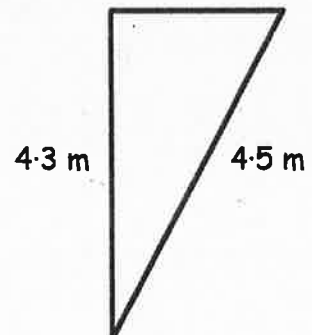
4. The diagram shows the base of a loud speaker stand.
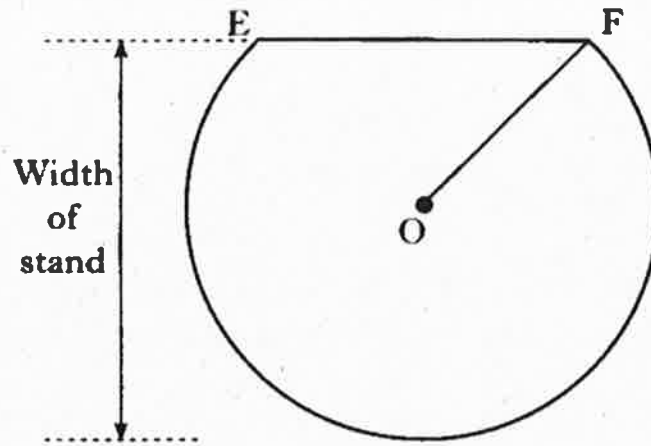
- The centre of the circle is O.
- EF is a chord of the circle with length 18cm.
- The radius, OF, of the circle is 15 cm.

   Find the width of the stand.

5. The diagram shows the cross section of a tunnel.
   It consists of a circle with a horizontal base.

   The radius of the circle is 1·95 metres and the width of the base is 2·5 metres.
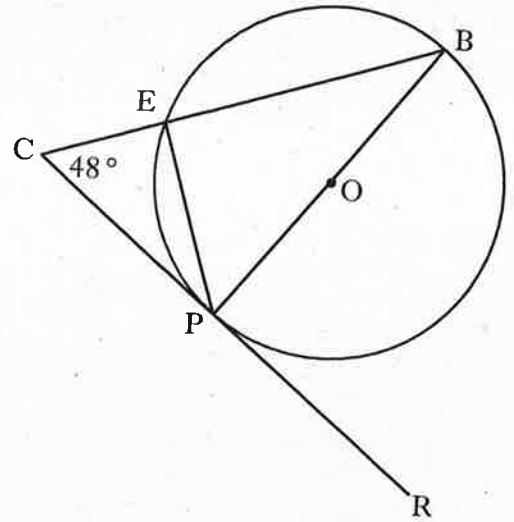   For safety reasons the tunnel must have a minimum height of 5·5 metres.
   Does the tunnel satisfy the safety regulations?
   Give a reason for your answer.
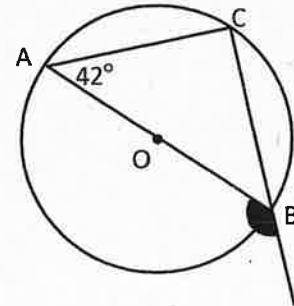
6. A circle, centre O, is shown.
   - PB is a diameter.
   - CR is a tangent to the circle at point P.
   - Angle BCP is 48°.

   Calculate the size of angle EPR.

7. The diagram shows a circle with centre O. AB is a diameter and C is a point on the circumference of the circle.
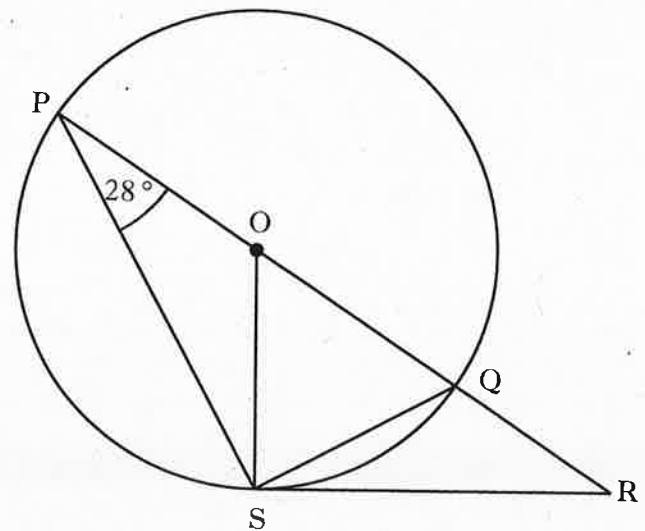
   Calculate the size of the shaded angle.

8. In the diagram,
   - O is the centre of the circle.
   - PQ is a diameter of the circle.
   - PQR is a straight line.
   - RS is a tangent to the circle at S.
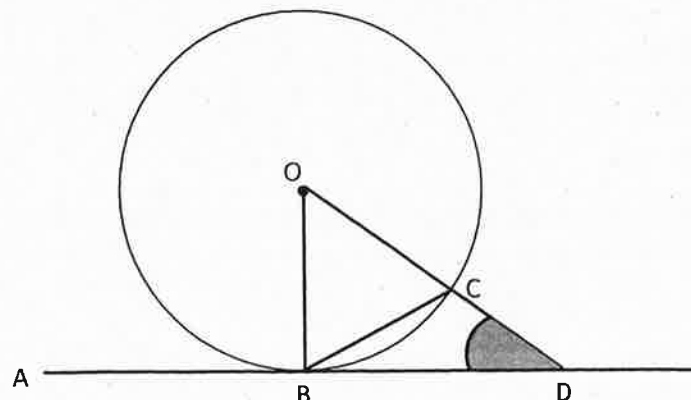   - Angle OPS is 28°.

   Calculate the size of angle QRS.

9. In the diagram, AD is a tangent to the circle at B. C lies on the circumference of the circle.
   Angle DBC = 34°.
   Calculate the size of angle CDB.
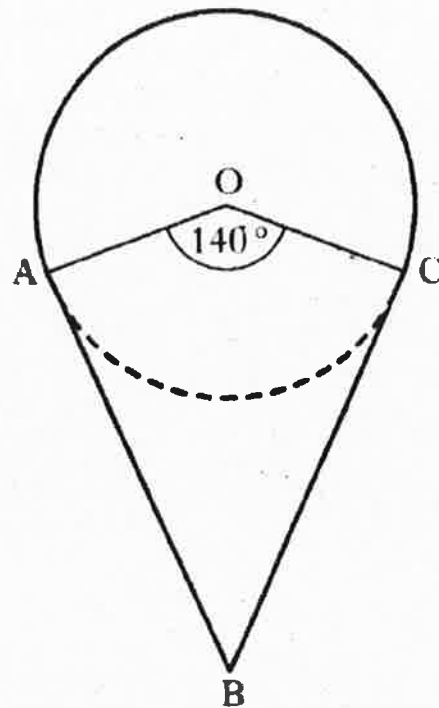
10. Kite OABC and a circle with centre O are shown.
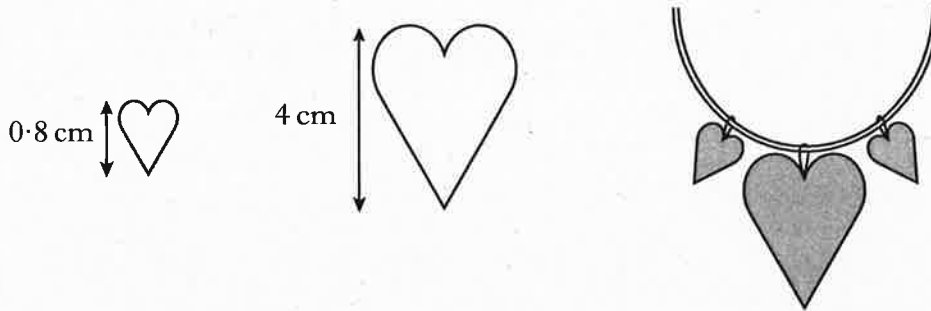AB is a tangent to the circle at A.
BC is a tangent to the circle at C.
Given that angle AOC = 140°, calculate angle ABC,

11. A necklace is made of beads which are mathematically similar.
    The height of the smaller bead is 0·8cm and its area is 0·6cm².
    The height of the larger bead is 4 cm.
    Find the area of the larger bead.

0·8 cm

4 cm

12. Two fridge magnets are mathematically similar.
    One magnet is 4cm long and the other is 10cm long.
    The area of the smaller magnet is 18 square centimetres.
   Calculate the area of the larger magnet.

←—4 cm—→   ←————10 cm————→
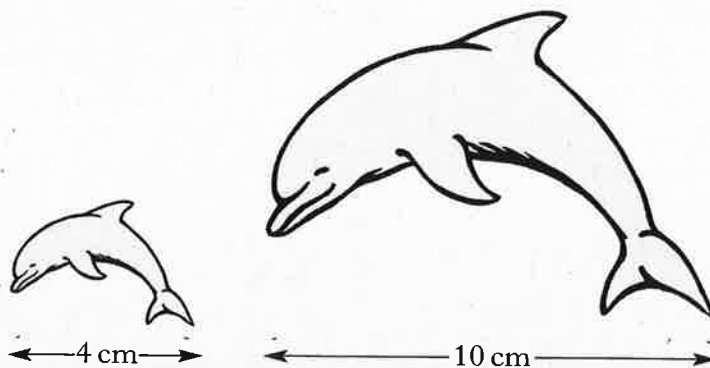
13. These wine glasses are similar in shape.
    The smaller glass holds 135ml of wine.
    How much will the larger glass hold?

12 cm   20 cm
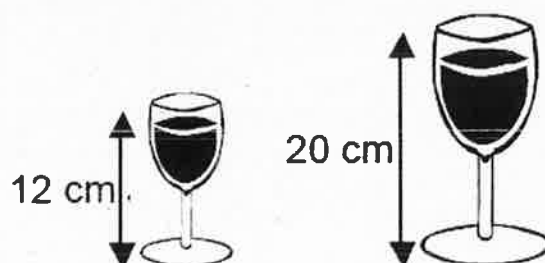
14. Two metal pipes are mathematically similar.
   The smaller pipe has a volume of 8000cm³.
   Find the volume of the larger pipe.

   20 cm

   40 cm
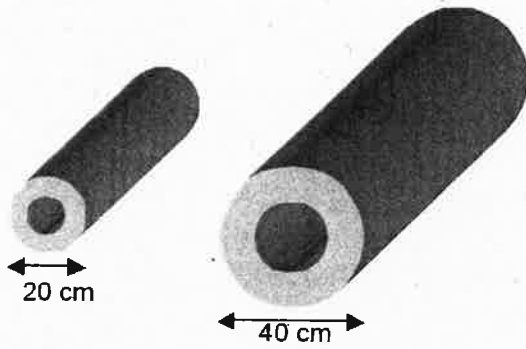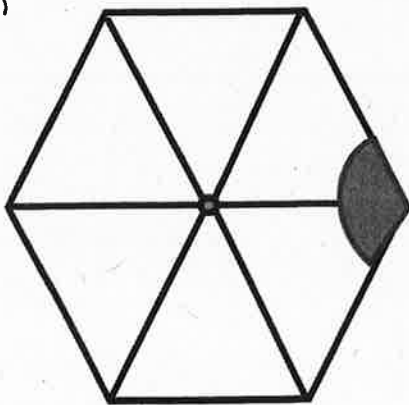
15. Two balloons are similar in shape.
   The larger balloon has a volume of 2187cm³.
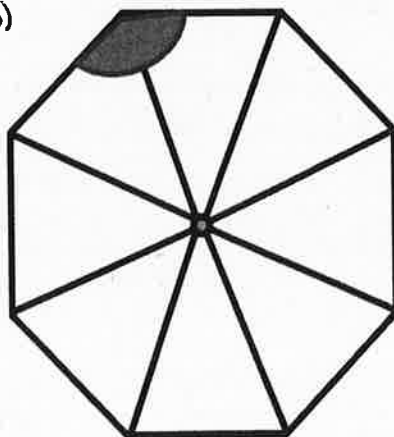   Calculate the volume of the smaller balloon.

   10 cm

   18 cm
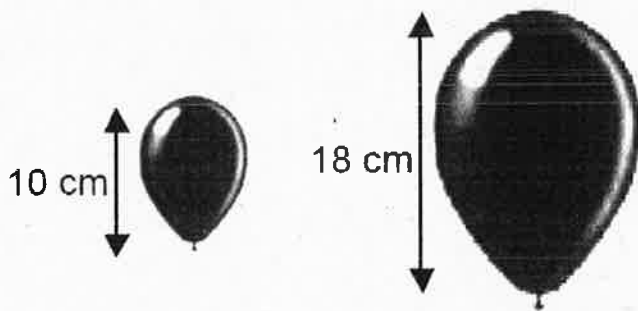
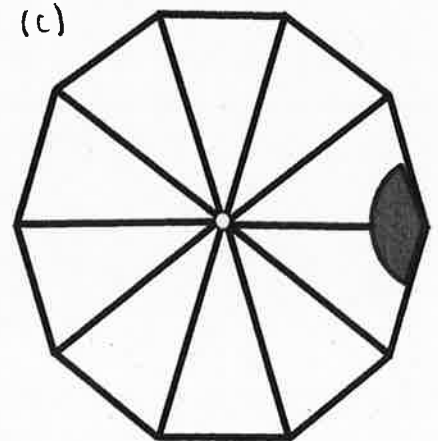16. Calculate the size of the shaded angle in each polygon.
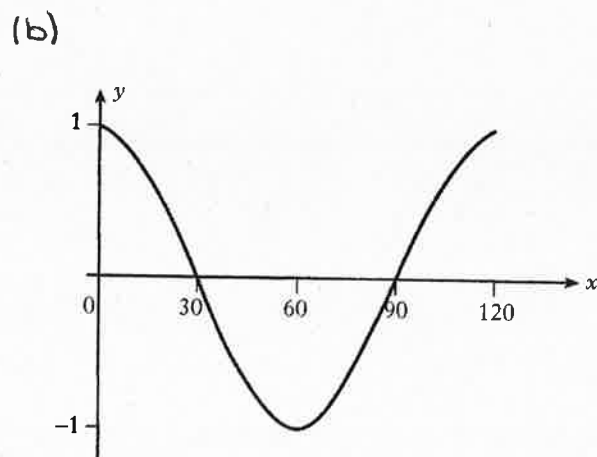
(a)

(b)

(c)

ASSESSMENT STANDARD 1.5

1. Sketch the graph of

    (a) $y = 2\sin x°$ for $0° \le x \le 360°$.
    (b) $y = 4\sin x°$ for $0° \le x \le 360°$.
    (c) $y = 2\cos x°$ for $0° \le x \le 360°$.
    (d) $y = 3\cos x°$ for $0° \le x \le 360°$.
    (e) $y = 6\sin x°$ for $0° \le x \le 360°$.
    (f) $y = 10\cos x°$ for $0° \le x \le 360°$.

2. Write down the period and amplitude of

    (a) $y = \sin 3x°$
    (b) $y = \cos 3x°$
    (c) $y = \sin 4x°$
    (d) $y = \cos 6x°$
    (e) $y = 2\sin 3x°$
    (f) $y = 4\cos 2x°$
    (g) $y = 6\sin 10x°$
    (h) $y = 3\sin 4x°$
    (i) $y = 7\cos 6x°$
    (j) $y = 3\sin 9x°$

3. The graphs below both have equations of the form $y = a\cos bx°$.
State the values of a and b.

(a)

(b)
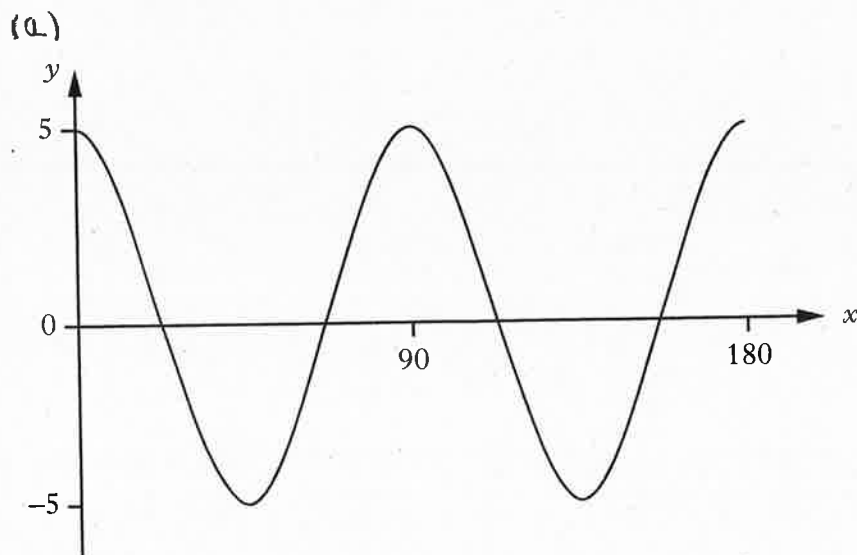
4. Solve the equations.

   (a) $10\sin x - 1 = 0$, $0° \le x \le 360°$.

   (b) $5\cos x + 2 = 0$, $0° \le x \le 360°$.

   (c) $6\sin x + 1 = 0$, $0° \le x \le 360°$.

   (d) $2\tan x - 7 = 0$, $0° \le x \le 360°$.

   (e) $10\cos x - 2 = 0$, $0° \le x \le 360°$.

   (f) $9\sin x + 6 = 2$, $0° \le x \le 360°$.
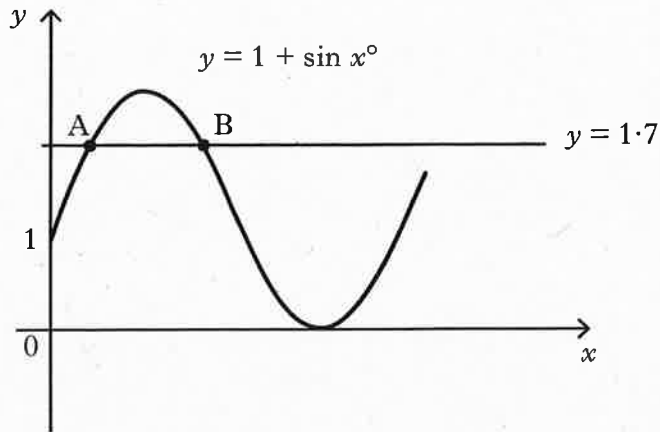
   (g) $3\cos x + 6 = 8$, $0° \le x \le 360°$.

5. The diagram shows the graph of $y = \cos x°$,
   The line $y = -0.5$ cuts the graph at B and C.
   Find the x coordinates of B and C.
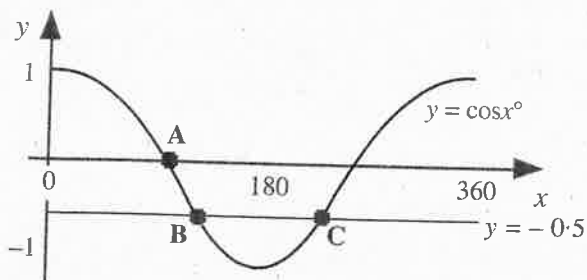
6. The diagram shows the graph of $y = 2\sin x°$.

   Write down the coordinates of the point P.

   The line $y = 1.5$ is drawn on the diagram.

   Find the x coordinate of the point Q.
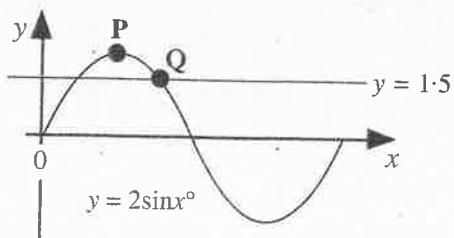
7. Part of the graph of y = 1 + sinx° is shown below.

$$y = 1 + \sin x^\circ$$

The line with equation y = 1·7 is drawn on the diagram.

It cute the graph of y = 1 + sinx° at A and B as shown.

Find the x coordinates of A and B.

ANSWERS

ASSESSMENT STANDARD 1.1

1. (a) $y = 2x - 3$ (b) $y = 3x + 5$ (c) $y = -3x + 4$

   (d) $y = -x - 6$ (e) $y = 4x - 9$ (f) $y = -4x - 14$

2. (a) $x < 4$ (b) $y \geq 3$ (c) $w > -4$

   (d) $k \leq -2$ (e) $x < 3$ (f) $x \geq -3$

   (g) $x < 8$ (h) $m \leq 4$

3. (a) $2f + 3s = 13$ (b) $3a + 4c = 75$ (c) $2a + 3c = 88$

4. (a) $x = 2, y = 3$ (b) $m = 1, n = -1$ (c) $a = 3, \ b = 4$

5. £9·70

6. £28

7. (a) $x = \dfrac{y-p}{2}$ (b) $x = \dfrac{p-u}{v}$ (c) $x = \dfrac{n}{2m}$

   (d) $x = \dfrac{8y+16}{7}$ (e) $x = \dfrac{5a-50}{6}$ (f) $x = \dfrac{M-3}{6}$

   (g) $x = \dfrac{h+12}{20}$ (h) $x = \dfrac{2d+6}{5}$ (i) $x = \dfrac{c-5}{4a}$

1. (a) k = 5   (b) k = 3   (c) k = 2   (d) k = -2

   (e) k = 6   (f) $k = -\dfrac{1}{2}$

2. (a) $y = (x - 4)^2 + 2$ (b) $y = (x + 3)^2 - 2$

3. (a) $y = (x - 3)^2 - 9$ (b) a = -3, b = 2   (c) a = 2, b = 16

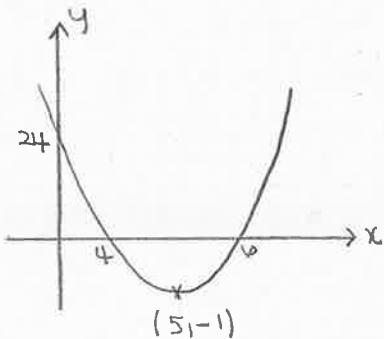4. (a) x = 4 (4, -3) minimum   (b) x = -2 (-2, 5) minimum

   (c) x = 2 (2, 3) minimum   (d) x = 1 (1, 3) minimum

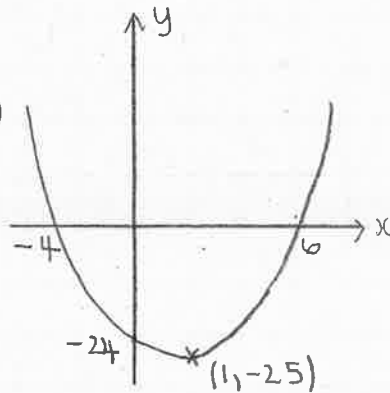   (e) x = -2 (-2, -6) minimum   (f) x = -3 (-3, 1) minimum

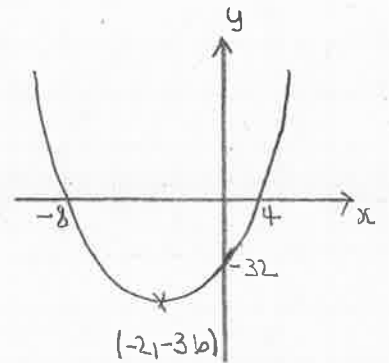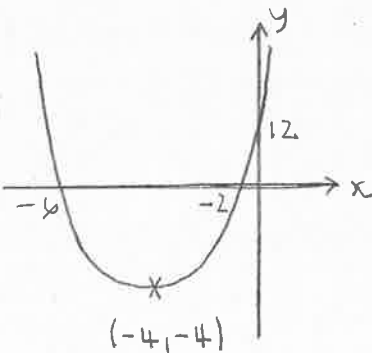   (g) x = -5 (-5, -8) minimum   (h) x = 6 (6, 3) minimum

5.

(a) vertex (5, -1), intercepts 4 and 6, y-intercept 24

(b) vertex (1, -25), intercepts -4 and 6, y-intercept -24

(c) vertex (-2, -36), intercepts -8 and 4, y-intercept -32

(d) vertex (-4, -4), intercepts -6 and -2, y-intercept 12

(e) vertex (2, -36), intercepts -4 and 8, y-intercept -32

(f) vertex (1, -16), intercepts -3 and 5, y-intercept -15

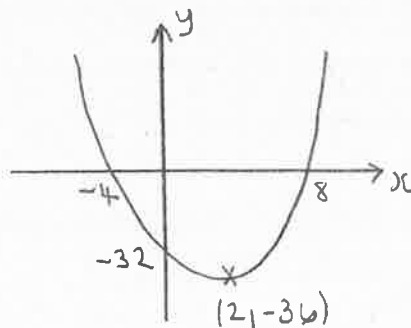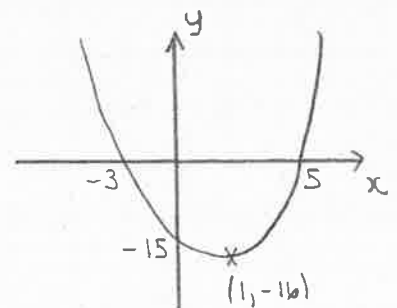## ASSESSMENT STANDARD 1.3

1. (a) $x = 5, x = 7$ (b) $x = 3, x = -2$ (c) $x = -1, x = 5$

  (d) $x = -4, x = -6$ (e) $x = 2, x = -6$ (f) $x = -6, x = -3$

  (g) $x = -2, x = 7$ (h) $x = 6, x = 1$

2. (a) $x = -0.46, -6.54$ (b) $x = 2.79, -1.79$ (c) $x = 10.48, -0.48$

  (d) $x = -0.88, -5.12$

3. Hits ground when h = 0   Quadratic formula gives $t = -0.19$, $t = 2.69$

   Time can't be negative, so hits ground after 2.69 seconds.

4. Hits ground when h = 0   Quadratic formula gives $t = -0.70, t = 10.7$

   Time can't be negative, so hits ground after 10.7 seconds.

5. Hits ground when h = 0   Quadratic formula gives $t = -0.37, t = 6.37$

   Time can't be negative, so hits ground after 6.37 seconds.

6. (a) 0   one real, repeated root   (b) -7   no real roots

  (c) 0   one real, repeated root   (d) 76   two real, distinct roots

  (e) 36   two real, distinct roots   (f) -19   no real roots

  (g) 25   two real, distinct roots   (h) 41   two real, distinct roots

  (i) 0   one real, repeated root   (j) -11   no real roots

1. $18·6^2 + 24·8^2 \neq 31·2^2$ By the converse of Pythagoras, the triangle is not right angled.

2. $15·6^2 + 11·7^2 = 19·5^2$ By the converse of Pythagoras, the triangle is right angled.

                                   He has been successful.

3. $1·6^2 + 4·3^2 \neq 4·5^2$ By the converse of Pythagoras, the triangle is not right angled.

4. 27 cm

5. Height of tunnel = 3·45 m.

    Height of tunnel is less than 5m so does not satisfy safety regulations.

6. 138°

7. 132°

8. 34°
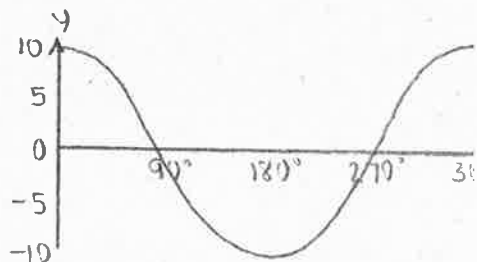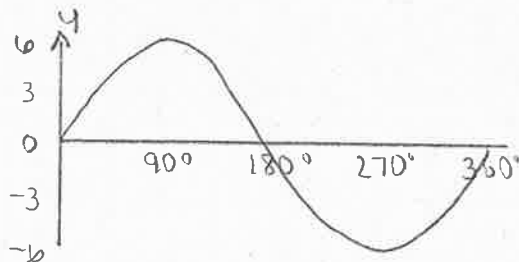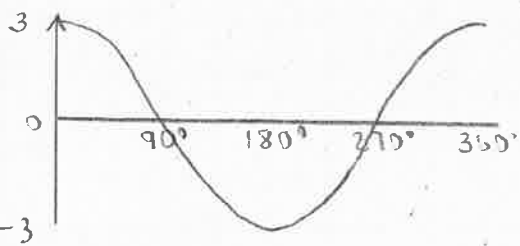
9. 22°

10. 40°
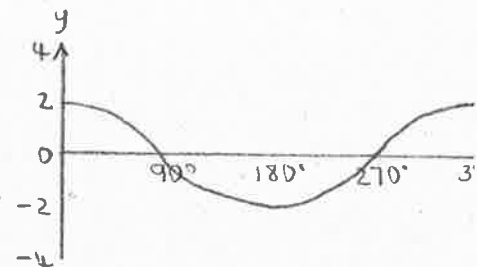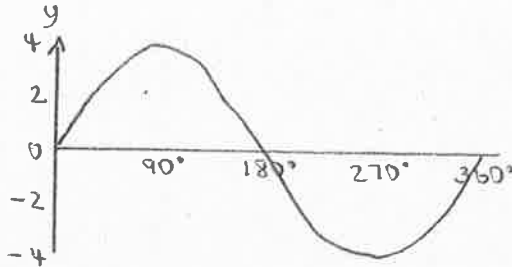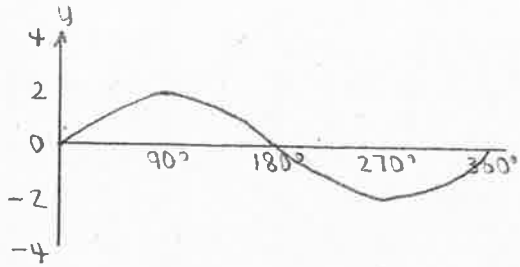
11. $15cm^2$

12. $112·5cm^2$

13. 625ml

14. $64000cm^3$

15. $375cm^3$

16. (a) 120°   (b) 135°   (c) 144°

1.

2. (a) period = 120° amplitude = 1   (b) period = 120° amplitude = 1

    (c) period = 90° amplitude = 1   (d) period = 60° amplitude = 1

    (e) period = 120° amplitude = 2   (f) period = 180° amplitude = 4

    (g) period = 36° amplitude = 6   (h) period = 90° amplitude = 3

    (i) period = 60° amplitude = 7   (j) period = 40° amplitude = 3

3. (a) a = 5, b = 4   (b) a = 1, b = 3

4. (a) 5·74°, 174·26°   (b) 113·58°, 246·42°   (c) 189·59°, 350·41°

    (d) 74·05°, 254·05°   (e) 78·46°, 281·54°   (f) 206·39°, 333·61°

    (g) 48·19°, 311·81°

5. x coordinate of B = 120°   x coordinate of C = 240°

6. 131·41°

7. x coordinate of A = 44·43°   x coordinate of B = 135·57°