National 5 Expressions and Formulae Extended Assessment Revision

Multiplying Out Brackets and Factorising

1. Multiply out the brackets and collect like terms:
   
   (a) \((x - 3)(x^2 + 4x - 1)\)  
   (b) \((x + 4)(2x^2 + 3x - 1)\)  
   (c) \((x + 3)(x^2 + 4x - 12)\)  
   
   (d) \((2y - 3)(y^2 + 4y - 1)\)  
   (d) \((3x + 1)(x^2 - 5x + 4)\)  
   (f) \((3x - 2)(2x^2 + x + 5)\)

2. Factorise:

   (a) \(x^2 + 2x - 15\)  
   (b) \(y^2 + y - 6\)  
   (c) \(x^2 - 7x - 8\)  
   (d) \(2p^2 - 5p - 12\)  
   
   (e) \(2x^2 + 3x - 2\)  
   (f) \(2x^2 + x - 6\)  
   (g) \(35 - 2q - q^2\)  
   (h) \(24 - 11s + s^2\)  
   
   (i) \(6m^2 + 13m + 5\)  
   (j) \(10d^2 - 11d - 6\)  
   (k) \(9p^2 + 18p - 16\)  
   (l) \(20 - 15a - 5a^2\)

3. Factorise:

   (a) \(x^2 - 25\)  
   (b) \(a^2 - 1\)  
   (c) \(a^2 - 100\)  
   
   (d) \(4p^2 - 9\)  
   (e) \(64p^2 - 121\)  
   (f) \(36 - 25u^2\)  
   
   (g) \(x^2 - 16y^2\)  
   (h) \(49t^2 - 144s^2\)  
   (i) \(f^2 - 900g^2\)  
   
   (j) \(5x^2 - 500\)  
   (k) \(3w^2 - 243\)  
   (l) \(10v^2 - 40\)  
   
   (m) \(12p^2 - 3\)  
   (n) \(20 - 45s^2\)  
   (o) \(7y^2 - 28z^2\)  
   
   (p) \(27a^2 - 48b^2\)  
   (q) \(125d^2 - 45e^2\)  
   (r) \(98f^2 - 200g^2\)

4. Write each expression in the form \((x + p)^2 + q:\)

   (a) \(x^2 + 6x + 10\)  
   (b) \(y^2 - 2y + 3\)  
   
   (c) \(z^2 + 8z - 10\)  
   (d) \(a^2 - 10a - 5\)  
   
   (e) \(b^2 + 18b - 81\)  
   (f) \(c^2 - 40c + 1\)  
   
   (g) \(r^2 + 5r - 5\)  
   (h) \(s^2 + s + 2\)  
   
   (i) \(t^2 - 3t - 1\)  
   (j) \(m^2 + \frac{1}{2}m + \frac{1}{4}\)  
   
   (k) \(n^2 + 0.6n - 1\)  
   (l) \(w^2 - 1.6w + 2\)
### Surds and Indices

1. **Express in simplest form**

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<tbody>
<tr>
<td>(a)</td>
<td>$\sqrt{12}$</td>
<td>(b)</td>
<td>$\sqrt{20}$</td>
<td>(c)</td>
</tr>
<tr>
<td>(e)</td>
<td>$\sqrt{45}$</td>
<td>(f)</td>
<td>$\sqrt{48}$</td>
<td>(g)</td>
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<td>(i)</td>
<td>$\sqrt{75}$</td>
<td>(j)</td>
<td>$\sqrt{44}$</td>
<td>(k)</td>
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<td>(m)</td>
<td>$5\sqrt{8}$</td>
<td>(n)</td>
<td>$3\sqrt{18}$</td>
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2. **Simplify**

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<tbody>
<tr>
<td>(a)</td>
<td>$\sqrt{3} \times \sqrt{3}$</td>
<td>(b)</td>
<td>$\sqrt{7} \times \sqrt{7}$</td>
<td>(c)</td>
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<tr>
<td>(d)</td>
<td>$\sqrt{4} \times \sqrt{3}$</td>
<td>(e)</td>
<td>$\sqrt{9} \times \sqrt{2}$</td>
<td>(f)</td>
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<td>(g)</td>
<td>$\sqrt{2} \times \sqrt{5}$</td>
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<td>(j)</td>
<td>$\sqrt{2} \times \sqrt{8}$</td>
<td>(k)</td>
<td>$\sqrt{12} \times \sqrt{3}$</td>
<td>(l)</td>
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<tr>
<td>(m)</td>
<td>$\sqrt{2} \times \sqrt{10}$</td>
<td>(n)</td>
<td>$\sqrt{3} \times \sqrt{6}$</td>
<td>(o)</td>
</tr>
<tr>
<td>(p)</td>
<td>$3\sqrt{2} \times 5\sqrt{2}$</td>
<td>(q)</td>
<td>$3\sqrt{2} \times 5\sqrt{2}$</td>
<td>(r)</td>
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3. **Rationalise the denominators of these fractions:**

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<tbody>
<tr>
<td>(a)</td>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>(b)</td>
<td>$\frac{1}{\sqrt{5}}$</td>
<td>(c)</td>
<td>$\frac{6}{\sqrt{3}}$</td>
<td>(d)</td>
</tr>
<tr>
<td>(f)</td>
<td>$\frac{10}{\sqrt{5}}$</td>
<td>(g)</td>
<td>$\frac{7}{\sqrt{3}}$</td>
<td>(h)</td>
<td>$\frac{3}{\sqrt{5}}$</td>
<td>(i)</td>
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There are some rules you need to remember when using indices:

\[ a^m \times a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \]

\[ (a^m)^n = a^{mn} \quad (ab)^m = a^m b^m \]

\[ a^{-m} = \frac{1}{a^m} \quad \sqrt[n]{a} = a^{\frac{1}{n}} \]

\[ \sqrt[n]{a^m} = a^{\frac{m}{n}} \quad a^0 = 1 \quad a^1 = a \]

1. Simplify each expression

(a) \( a^5 \times a^4 \) \quad (b) \( n^{-12} \times n^9 \) \quad (c) \( c^6 \times c \) \quad (d) \( \frac{1}{d^2} \times d^3 \)

(e) \( 3a^4 \times 5a^3 \) \quad (f) \( 4b^9 \times 2b^{-6} \) \quad (g) \( 8c^8 \times 7c \) \quad (h) \( \frac{v^6}{v^2} \)

(i) \( y^{19} \div y^{-5} \) \quad (j) \( \frac{k8}{k} \) \quad (k) \( \frac{12c^5}{6c^3} \) \quad (l) \( \frac{48f^{10}}{6f^{-4}} \)

(m) \( 30c^6 \div c^4 \) \quad (n) \( (c^6)^5 \) \quad (o) \( (y^7)^{-5} \) \quad (p) \( (6h^5)^3 \)

(q) \( (2x^{-2})^5 \) \quad (r) \( (xy)^5 \) \quad (s) \( (x^2y^3)^4 \) \quad (t) \( (h^3k^5)^{-8} \)
2. Express in its simplest form:

(a) \( k^8 \times \left( k^2 \right)^3 \)  
(b) \( 6x^\frac{3}{2} + 2x^\frac{1}{2} \)  
(c) \( \frac{\frac{1}{2} \times b^\frac{3}{2}}{b} \)  
(d) \( \frac{3a^5 \times 2a}{a^2} \)  

(e) \( a^2 \left( 2a^{-\frac{1}{2}} + a \right) \)  
(f) \( a^\frac{2}{3} \left( a^\frac{2}{3} - a^{-\frac{2}{3}} \right) \)  
(g) \( a^\frac{1}{3} \left( a^{-\frac{1}{3}} - 2 \right) \)  
(h) \( x^\frac{1}{3} \left( 3x + x^{-\frac{2}{3}} \right) \)

Algebraic Fractions

1. Express as a single fraction

(a) \( \frac{2}{a + 1} + \frac{3}{a} \)  
(b) \( \frac{4}{w + 3} - \frac{7}{w} \)  
(c) \( \frac{6}{e - 5} + \frac{9}{e} \)  

(d) \( \frac{5}{m} - \frac{8}{m + 2} \)  
(e) \( \frac{1}{r} + \frac{9}{r - 7} \)  
(f) \( \frac{7}{v} - \frac{3}{v - 9} \)  

(g) \( \frac{3}{b + 1} + \frac{2}{b + 3} \)  
(h) \( \frac{4}{n - 5} + \frac{6}{n + 2} \)  
(i) \( \frac{5}{s + 4} - \frac{8}{s + 7} \)  

(j) \( \frac{7}{t - 2} - \frac{9}{t + 5} \)  
(k) \( \frac{5}{x - 1} + \frac{1}{x - 7} \)  
(l) \( \frac{9}{y - 8} - \frac{6}{y - 5} \)

Circle: Length of an Arc and Area of a Sector

1.

The diagram below shows a sector of a circle, centre C.

The radius of the circle is 7.3 centimetres and angle PCR is 54°.
Calculate the area of the sector PCR.
2. A pet shop manufactures protective dog collars. In the diagram below the shaded area represents one of these collars.

\[ \text{AB and CD are arcs of the circles with centres at O.} \]
\[ \text{The radius, OA, is 10 inches and the radius, OC, is 18 inches.} \]
\[ \text{Angle AOB is } 160^\circ. \]

Calculate the area of a collar.

3. The diagram below shows a sector of a circle, centre C.

\[ \text{The radius of the circle is 5 centimetres and angle ACB is } 72^\circ. \]
\[ \text{Calculate the length of arc AB.} \]
\[ \textbf{Take } \pi = 3.14. \]
4. The diagram below shows a circle, centre C.

The circumference of the circle is 40.8 centimetres.
Calculate the length of the minor arc AB.

Volume of Solids

1. Work out the volume of each of these solids.
2. A health food shop produces cod liver oil capsules for its customers. Each capsule is in the shape of a cylinder with hemispherical ends as shown in the diagram below.

![Diagram of a capsule with dimensions 15 mm and 23 mm]

The total length of the capsule is 23 millimetres and the length of the cylinder is 15 millimetres. Calculate the volume of one cod liver oil capsule.

3. A company manufactures aluminium tubes. The cross-section of one of the tubes is shown in the diagram below.

![Diagram of a tube with dimensions 74 mm and 82 mm, and length 900 mm]

The inner diameter is 74 millimetres. The outer diameter is 82 millimetres. The tube is 900 millimetres long. Calculate the volume of aluminium used to make the tube.

*Give your answer correct to three significant figures.*
The Battle of Largs in 1263 is commemorated by a monument known as The Pencil. This monument is in the shape of a cylinder with a cone on top.

The cylinder part has diameter 3 metres and height 15 metres.

(a) Calculate the volume of the **cylinder** part of The Pencil.

The volume of the **cone** part of The Pencil is 5.7 cubic metres.

(b) Calculate the **total** height of The Pencil.
5. (a) A candle is in the shape of a cylinder with diameter 10 centimetres and height 15 centimetres.

Calculate the volume of the candle.
Give your answer correct to 3 significant figures.

(b) A second candle is in the shape of a cone with a circular base of diameter 14 centimetres and height $h$ centimetres.

It has the same volume as the first candle.
Calculate $h$. 