1. $\int_a^2 (x^2-1)\,dx = 0 \Rightarrow \left[\frac{x^3}{3} - x\right]_a^2 = 0$

$$\left(\frac{8}{3}-2\right) - \left(\frac{a^3}{3}-a\right) = 0$$

$$\frac{8-6}{3} - \frac{a^3}{3} + a = 0$$

$$\frac{2}{3} - \frac{a^3}{3} + a = 0$$

$$\Rightarrow a - \frac{a^3}{3} = -\frac{2}{3}$$

$$3a - a^3 = -2$$

$$a(3-a^2) = -2$$

$$a(a^2-3) = 2$$

$$a = -1$$

> By trial and error since $a < 2$, try $1, 0, -1$ and find that
> $-1((-1)^2 - 3)$
> $= -1(1-3)$
> $= -1(-2)$
> $= 2$

<u>Alternative Method</u>:

$$3a - a^3 = -2$$

By synthetic division $\Rightarrow 3a - a^3 + 2 = 0$

$\Rightarrow a^3 - 3a - 2 = 0$

$a^3 - 0a^2 - 3a - 2$

	1	0	−3	−2
−1		−1	1	2
	1	−1	−2	0

factors $(x+1)(x^2-x-2) \doteq (x+1)(x+1)(x-2)$

$x = -1$ or $x = 2$

Hence $a = -1$

2. $f(x) = 0 \Rightarrow x(x^2-3)(x^2+4)(x^2-1) = 0$

$x = 0, \quad x = \pm\sqrt{3}, \quad x = \pm 1$

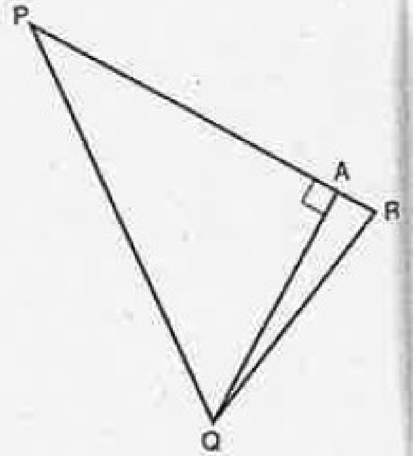
Note: $x^2 + 4 = 0 \Rightarrow x^2 = -4$ no real roots $x \notin R$

$s.s.\ \{-\sqrt{3}, -1, 0, \sqrt{3}, 1\}$

3. P(−2, 5), Q(2, −1), R(4, 2)

$m_{PR} = \dfrac{5-2}{-2-4} = \dfrac{3}{-6} = -\dfrac{1}{2}$

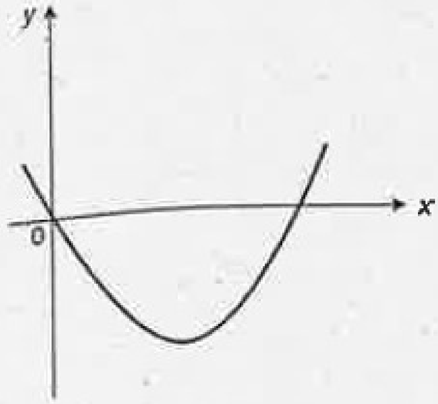
$m_{PR} \cdot m_{AQ} = -1$ if AQ $\perp$ PR



$m_1 m_2 = -1 \qquad m_1 = -\dfrac{1}{2},\ m_2 = 2$, through Q(2, −1)

$$y + 1 = 2(x-2)$$
$$y + 1 = 2x - 4$$
$$y = 2x - 5 \text{ is equation of altitude.}$$

4.



Graph of $6 + x - x^2$

(i) coefficient of x^2 negative $\Rightarrow$ shape

(ii) $ax^2 + bx + c$ cuts y at $(0, c)$
$c = 6$, point (0, 6)

2 main reasons why graph is not $f(x)$

$f(x) = ax^2 + bx + c$

$a < 0$ parabola should be inverted 

c +ve $(0, c)$ should cut y at (0, 6)

5. General equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 + 12x - 4y + 15 = 0$

$2g = 12,\ -g = -6,\ 2f = -4,\ -f = 2$

Centre $= (-g, -f)$, $r = \sqrt{g^2 + f^2 - c}$

Centre $= (-6, 2)$ $r = \sqrt{6^2 + 2^2 - 15} = \sqrt{25} = 5.$

Hence circle has centre (−6, 2) radius 5.

6. $\log a + \log b = \log(ab)$; $\log a - \log b = \log\left(\frac{a}{b}\right)$

Hence $\log a + \log b - \log c = \log\left(\frac{ab}{c}\right)$

$\log_{10}40 + \log_{10}20 - \log_{10}80$

$= \log_{10}((40 \times 20) \div 80)$

$= \log_{10}10$

$= 1$

7. $x^3 - 3x^2 - x + a$, if divisible by $(x-3)$ then $f(3) = 0$ by synthetic division

	1	−3	−1	a
$x = 3$		3	0	−3
	1	0	−1	0

(Remainder = 0)

$\Rightarrow -3 + a = 0$

$\Rightarrow a = 3$

$a = 3$

$f(x) = x^3 - 3x^2 - x + 3$

$f(x) = (x-3)(x^2-1)$

$\Rightarrow f(x) = (x-1)(x+1)(x-3)$ fully factorised.

8. For function to have equal roots, discriminant = 0, discriminant = $b^2 - 4ac$

$b^2 - 4ac = 0$

$3x^2 - 2x - c$

$a = 3,\ b = -2,\ c = -c$

$b^2 - 4ac = 0 \Rightarrow (-2)^2 - 4(3)(-c) = 0$

$\Rightarrow 4 + 12c = 0$

$\Rightarrow 12c = -4$

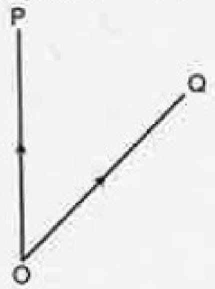
$\Rightarrow c = \frac{-4}{12} = -\frac{1}{3}$

Equation is

$3x^2 - 2x + \frac{1}{3}$ or $\frac{1}{3}(9x^2 - 6x + 1) = \frac{1}{3}(3x-1)^2$

9. $\overrightarrow{OP} = p$ $\overrightarrow{OQ} = q$

$P = (1, 4, -1),\ Q = (1, -2, 3)$



$p = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ $q = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

$p \cdot q = (1 \times 1) + (4 \times (-2)) + (-1 \times 3)$

$= 1 - 8 - 3 = -10$

$|p| = \sqrt{1^2 + 4^2 + (-1)^2}$

$= \sqrt{18}$

$|q| = \sqrt{1^2 + (-2)^2 + 3^2}$

$= \sqrt{14}$

$$\frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|p||q|} = \cos P\hat{O}Q$$

$$= \frac{-10}{\sqrt{18}\sqrt{14}}$$

10. (a) $u_{r+1} = mu_r + c,\ u_0 = 2,\ u_1 = -1,\ u_2 = 14$

$u_1 = m(u_0) + c \Rightarrow -1 = 2m + c$ ①

$u_2 = m(u_1) + c \Rightarrow 14 = -m + c$ ②

② − ① $15 = -3m \Rightarrow m = -5$

Substitute $m = -5$ in ①, $-1 = 2(-5) + c$

$-1 = -10 + c$

$9 = c$

$m = -5,\ c = 9,\ \Rightarrow u_{r+1} = -5u_r + 9$

(b) $u_3 = -5u_2 + 9$

$= -5(14) + 9$

$= -70 + 9$

$u_3 = -61$

To find u_{-1} use $u_0 = -5u_{-1} + 9,\ u_0 = 2$

$2 = -5u_{-1} + 9$

$-7 = -5u_{-1}$

$\frac{7}{5} = u_{-1}$

(c) To find $u_r = u_{r+1} \Rightarrow u_r = -5u_r + 9$

$\Rightarrow 6u_r = 9$

$u_r = \frac{9}{6}, = \frac{3}{2}$

Check $-5\left(\frac{3}{2}\right) + 9 = -\frac{15}{2} + \frac{18}{2} = \frac{3}{2}$

$(u_r, u_{r+1}) = \left(\frac{3}{2}, \frac{3}{2}\right)$

11. $f(x) = 3\sin(2x + 30°)$ let $x = 2x + 30°$

Sin x has maximum value 1 when $x = 90°$ maximum of $3\sin x = 3$

Sin x has minimum value −1 when $x = 270°$ minimum of $3\sin x = -3$

For maximum, $2x + 30 = 90$; $2x = 60$; $x = 30$

For minimum, $2x + 30 = 270$; $2x = 240$; $x = 120$

Period of the graph $= \frac{360}{2} = 180$ (add 180 to 30 and 120)

$f(x)$ has 2 maximum turning points (30°, 3) and (210°, 3)

$f(x)$ has 2 minimum turning points (120°, −3) and (300°, −3)

12. $2 \sin 2x + 1 = 0$

$2 \sin 2x = -1$

$\sin 2x = -\frac{1}{2}; = -0{\cdot}5$

$\sin^{-1}(0{\cdot}5) = 30$ in quadrant 1, sin is negative in quadrants 3 and 4

In quadrant 3 angle = 180 + 30

In quadrant 4 angle = 360 − 30

$2x = 210^\circ$ or 330°

$x = 105^\circ$ or 165°

Period of $\sin 2x = \frac{360}{2} = 180^\circ$ (i.e., 2 cycles in 360°) add 180 to each of 105 and 165.

Hence x has 4 possible values 105°, 165°, 285° and 345°.