1. If divisible by $x + 2$ then $f(-2) = 0$

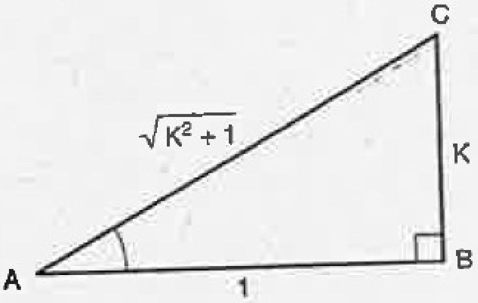
	x^3	$+\ 4x^2$	$+\ x$	$-\ t$
-2	1	4	1	$-t$
		-2	-4	6
	1	2	-3	$6 - t$
$t = 6, \Rightarrow$	1	2	-3	0

$6 - t \Rightarrow 6 - t = 0 \Rightarrow t = 6$

$f(x) = (x+2)(x^2 + 2x - 3)$

$f(x) = (x+2)(x+3)(x-1)$ fully factorised.

2. By Pythagoras' theorem



$K^2 + 1^2 = |AC|^2$

$AC = \sqrt{K^2+1}$

$\cos A = \dfrac{1}{\sqrt{K^2+1}}$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 2\left(\dfrac{1}{\sqrt{K^2+1}}\right)^2 - 1$

$= \dfrac{2}{K^2+1} - 1, = \dfrac{2 - 1(K^2+1)}{K^2+1}$

Hence $\cos 2A = \dfrac{2 - K^2 - 1}{K^2+1} = \dfrac{1-K^2}{1+K^2}$

3 *(a)* General equation of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$

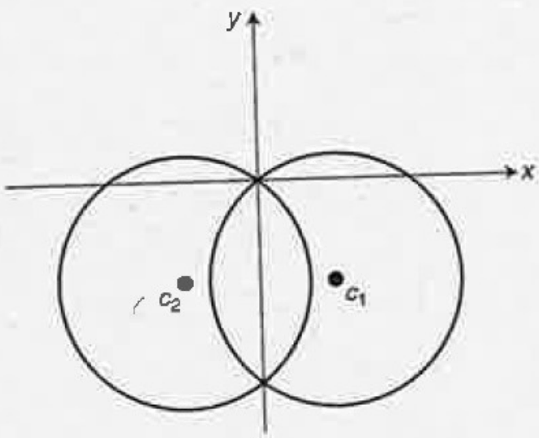
centre $= (-g, -f)$, radius $= \sqrt{g^2 + f^2 - c}$

$x^2 + y^2 - 6x + 8y = 0$.

centre $= (3, -4)$, $r = \sqrt{3^2 + (-4)^2} = 5.$

centre $(3, -4)$, radius $= 5.$

(b)



reflected in y-axis $c_1 \rightarrow c_2 \Rightarrow c_2 = (-3, -4)$

radius unchanged

new equation $= x^2 + y^2 + 6x + 8y = 0$

4.

$$\frac{2+x}{2} - (2-x) < 5$$

$$2 + x - 2(2-x) < 10$$

$$2 + x - 4 + 2x < 10$$

$$3x - 2 < 10$$

$$3x < 12$$

$$x < 4$$

5.

$$f(x) = 3\sin 2x$$

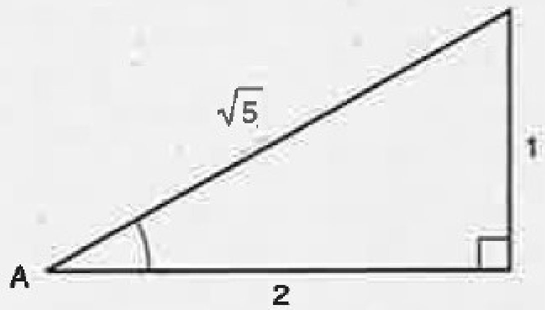
$$f'(x) = 6\cos 2x$$

$$x = \frac{\pi}{6} \Rightarrow 6\cos 2\left(\frac{\pi}{6}\right)$$

$$= 6\cos\left(\frac{\pi}{3}\right) = 6 \times \frac{1}{2} = 3$$

$$f'\left(\frac{\pi}{6}\right) = 3$$

6. By Pythagoras' Theorem $\sqrt{2^2 + 1^2}$ = hypotenuse $= \sqrt{5}$ hypotenuse.



Hence cos A $= \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$= p\sqrt{5}$ where $p = \frac{2}{5}$

7.

$$f(x) = (2x + \sqrt{x})^3$$

$$f(x) = (2x + x^{1/2})^3 \qquad \text{(By chain rule method.)}$$

$$f'(x) = 3(2x + x^{1/2})^2 \left(2 + \frac{1}{2}x^{-1/2}\right)$$

$$= 3\left(2 + \frac{1}{2\sqrt{x}}\right)(2x + \sqrt{x})^2$$

Hence $$f'(x) = 3\left(2 + \frac{1}{2\sqrt{x}}\right)(2x + \sqrt{x})^2$$

and $$f'(4) = 3\left(2 + \frac{1}{2\sqrt{4}}\right)(2(4) + \sqrt{4})^2$$

$$= 3(2\tfrac{1}{4})(10)^2$$

$$= 3(225)$$

$$= 675$$

8. $f(x) = x(x+2)(x^2-3)(x^2+1)(x^2-4) = 0$

Note: $x^2 + 1 = 0, \Rightarrow x^2 = -1$, (not real) $x \notin R$

$\Rightarrow f(x) = 0, x = 0, x = -2, x = \pm\sqrt{3}, x = \pm 2$

Hence, S.S. $\{-\sqrt{3}, -2, 0, \sqrt{3}, 2\}$

9. $h(x) = g(f(x)) \quad g(x) = -x^2 + x + 2 \quad f(x) = 2x - 1$

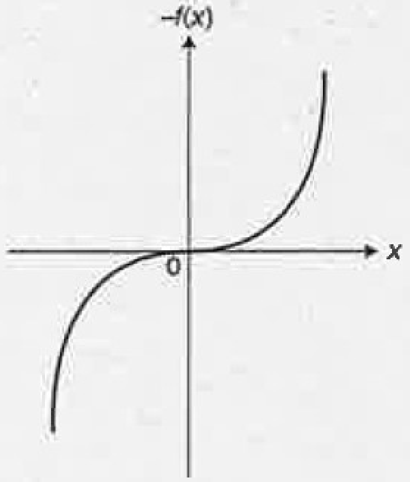
$$g(f(x)) = g((2x-1)) = -(2x-1)^2 + (2x-1) + 2$$

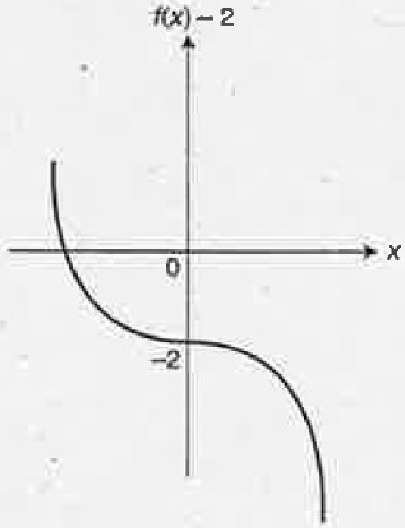
$$= -(4x^2 - 4x + 1) + 2x - 1 + 2$$

$$= -4x^2 + 4x - 1 + 2x + 1$$

$$= -4x^2 + 6x$$

10.





11. $\cos BAC = \dfrac{AB \cdot AC}{|AB||AC|}$

$AB = \boldsymbol{b} - \boldsymbol{a}$

$AC = \boldsymbol{c} - \boldsymbol{a}$

$$\underset{\boldsymbol{b}}{\begin{bmatrix} -1 \\ 5 \\ 2 \end{bmatrix}} - \underset{\boldsymbol{a}}{\begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \qquad \underset{\boldsymbol{c}}{\begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}} - \underset{\boldsymbol{a}}{\begin{bmatrix} -2 \\ 3 \\ 4 \end{bmatrix}} = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

$BA \cdot BC = (1 \times 2) + (2 \times -2) + (-2 \times -1) = 2 + (-4) + (-2) = -4$

$|AB|^2 = 1^2 + 2^2 + (-2)^2 = 9; \qquad |AB| = 3$

$|AC|^2 = 2^2 + (-2)^2 + 1^2 = 9; \qquad |AC| = 3$

$\cos BAC = \dfrac{AB \cdot AC}{|AB||AC|} = \dfrac{-4}{3 \times 3} \doteq \dfrac{-4}{9}$ as given.

Since the cosine of angle BAC is negative, the angle is in the second quadrant and is an obtuse angle.

12. (a) $3x^2 - 6x + 5 = 3\left(x^2 - 2x + \frac{5}{3}\right) = 3\left((x^2 - 2x + 1) - 1 + \frac{5}{3}\right)$

$= 3\left[(x-1)^2 - \frac{3}{3} + \frac{5}{3}\right]$

$= 3\left[(x-1)^2 + \frac{2}{3}\right]$

$= 3(x-1)^2 + 2$

<u>Alternative Method</u>

$3x^2 - 6x + 5 = 3(x^2 - 2x) + 5 = 3[(x^2 - 2x + 1) - 1] + 5$

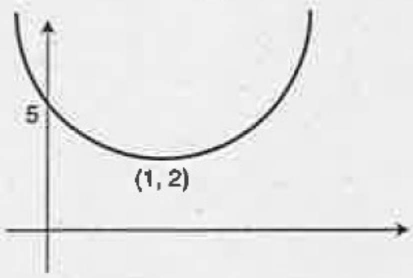
$3[(x-1)^2 - 1] + 5$

$3(x-1)^2 - 3 + 5$

$= 3(x-1)^2 + 2$

Since the least value of any square number is 0,
then $(x-1)^2$ has minimum value 0.
Hence the minimum value of the function is $0 + 2$ when $x = 1$.
Minimum value of the function is $3(1-1)^2 + 2$ and the coordinates of the minimum turning point $(1, 2)$.

(b) y-intersection $(0, 5)$, minimum turning point $(1, 2)$



(c) Since the minimum turning point $(1, 2)$ lies above the x-axis, the curve does not cross the x-axis hence the equation has no real roots.