1. $x^2 + 100 = x^2 - 100 + 200$

$$\frac{x^2 + 100}{x + 10} = \frac{(x + 10)(x - 10) + 200}{x + 10} = x - 10 + \frac{200}{x + 10}$$

$$k = 200$$

2. If the points are collinear, then lines are parallel.

Hence $\overrightarrow{AB} = k\overrightarrow{BC}$

A(3, –1), B(a, 2), C(b, 5)

$$a = \begin{pmatrix} 3 \\ -1 \end{pmatrix},\ b = \begin{pmatrix} a \\ 2 \end{pmatrix},\ c = \begin{pmatrix} b \\ 5 \end{pmatrix}$$

$$\overrightarrow{AB} = b - a = \begin{pmatrix} a - 3 \\ 3 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} b - a \\ 3 \end{pmatrix}$$

$\Rightarrow \quad a - 3 = b - a$

$\Rightarrow \quad 2a = b + 3$

$\Rightarrow \quad 2a - b = 3$

<u>Alternative Method</u>

A(3, –1), B(a, 2), C(b, 5)

If points are collinear then gradient of AB = gradient of BC.

$$m_{AB} = \frac{3}{a - 3}\ .\ m_{BC} = \frac{3}{b - a}$$

$$\Rightarrow \quad \frac{3}{a - 3} = \frac{3}{b - a}$$

$\Rightarrow \quad a - 3 = b - a$

$\Rightarrow \quad 2a = 3 + b$

$\Rightarrow \quad 2a - b = 3$

3. $K = 3\cos\left(3x - \frac{\pi}{2}\right), \qquad 0 \leq x \leq 2\pi$

maximum value = 3 when $\cos\left(3x - \frac{\pi}{2}\right) = 1$

$\cos 0 = 1,\ \cos 2\pi = 1,\ \cos 4\pi = 1$

$$\Rightarrow \quad \left(3x - \frac{\pi}{2}\right) = 0,\ 2\pi \text{ or } 4\pi$$

$$\Rightarrow \quad 3x = 0 + \frac{\pi}{2},\ 2\pi + \frac{\pi}{2} \text{ or } 4\pi + \frac{\pi}{2}$$

$$\Rightarrow \quad 3x = \frac{\pi}{2},\ \frac{5\pi}{2} \text{ or } \frac{9\pi}{6}$$

$$\Rightarrow \quad x = \frac{\pi}{6},\ \frac{5\pi}{6} \text{ or } \frac{3\pi}{2}$$

minimum value $= -3$ when $\cos\left(3x - \frac{\pi}{2}\right) = -1$

$\cos \pi = -1,\ \cos 3\pi = -1,\ \cos 5\pi = -1$

$\Rightarrow \left(3x - \frac{\pi}{2}\right) = \pi,\ 3\pi,\ 5\pi$

$\Rightarrow \quad 3x = \pi + \frac{\pi}{2},\ 3\pi + \frac{\pi}{2},\ 5\pi + \frac{\pi}{2}$

$\Rightarrow \quad 3x = \frac{3\pi}{2},\ \frac{7\pi}{2},\ \frac{11\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{2},\ \frac{7\pi}{6}\ \ \frac{11\pi}{6}$

solution set $\left\{\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$

or coordinates are $\left(\frac{\pi}{6}, +3\right), \left(\frac{\pi}{2}, -3\right), \left(\frac{5\pi}{6}, +3\right), \left(\frac{7\pi}{6}, -3\right), \left(\frac{3\pi}{2}, +3\right), \left(\frac{11\pi}{6}, -3\right)$

4. $2 \cos 2x - 1 = 0$

$2 \cos 2x = 1$

$\cos 2x = \frac{1}{2}$

$= 0{\cdot}5$

$\cos^{-1} (0{\cdot}5) =$ 60 in quadrant 1 and (360 – 60) in quadrant 4

$2x = 60°$ or $300°$

$x = 30°$ or $150°$

Period of $\cos 2x = \frac{360}{2} = 180°$(i.e., 2 cycles in 360°) add 180 to 30 and 150.

Hence, x has 4 possible values, 30°, 150°, 210° and 330°.

5. $f(x) = 0 \Rightarrow x(x^2 + 4)(x^2 - 3)(x^2 - 1) = 0$

$\Rightarrow x = 0,\ x = \pm\sqrt{3},\ x = \pm\sqrt{1}$

S.S. $\{-\sqrt{3}, -1, 0, 1, \sqrt{3}\}$

Note: $x^2 + 4 = 0 \Rightarrow x^2 = -4$

No real solution, $x \notin R$.

6. $f(x) = x^3 + 3x^2 - 4x + q$

If $(x - 2)$ is a factor of $f(x)$

Then $f(2) = 0$

By synthetic division

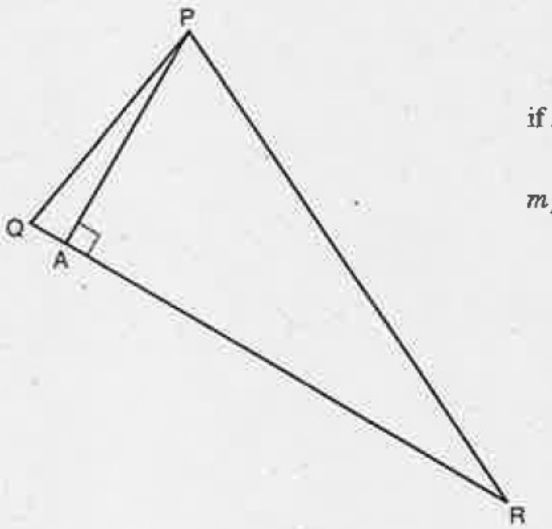
	x^3	$+\ 3x^2$	$-\ 4x$	$+\ q$
	1	3	−4	q
2		2	10	12
	1	5	6	$12 + q$

$12 + q \Rightarrow 12 + q = 0,$

$q = -12$

$f(x) = x^3 + 3x^2 - 4x - 12$

7. P(–1, 5), Q(–3, 2), R(9, –1)



if AP ⊥ to QR

$m_{AP} \cdot m_{QR} = -1$

$$m_{QR} = \frac{2-(-1)}{-3-9} = \frac{3}{-12} = \frac{-1}{4}$$

$m_{AP} = 4$ through P(–1, 5)

$y - 5 = 4(x + 1)$

$y - 5 = 4x + 4 \Rightarrow y = 4x + 9$ equation of altitude AP.

8. *(a)* $u_{r+1} = Ku_r + t,\ u_0 = 2,\ u_1 = -2,\ u_2 = 10$

$u_1 = Ku_0 + t \Rightarrow -2 = (2)K + t$ ①

$u_2 = Ku_1 + t \Rightarrow 10 = (-2)K + t$ ②

Add $\Rightarrow 8 = 2t,\ t = 4$

Substitute $t = 4$ in ①, $-2 = 2K + t \Rightarrow 2K + 4$

$\Rightarrow 2K = -6$

$K = -3$

$u_{r+1} = Ku_r + t,\ K = -3,\ t = 4$

$u_{r+1} = -3u_r + 4$

(b) To find $u_{r+1} = u_r \Rightarrow -3u_r + 4 = u_r$

$\Rightarrow 4u_r = 4$

$\Rightarrow u_r = 1$

Test $u_1 = 1,\ u_{r+1} = -3(1) + 4 = 1$

9. $f(x) = 3\cos 2x$ max $= 3$ when $2x = 0, 2\pi, 4\pi$

$x = 0, \pi, 2\pi$

min $= -3$ when $2x = \pi, 3\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

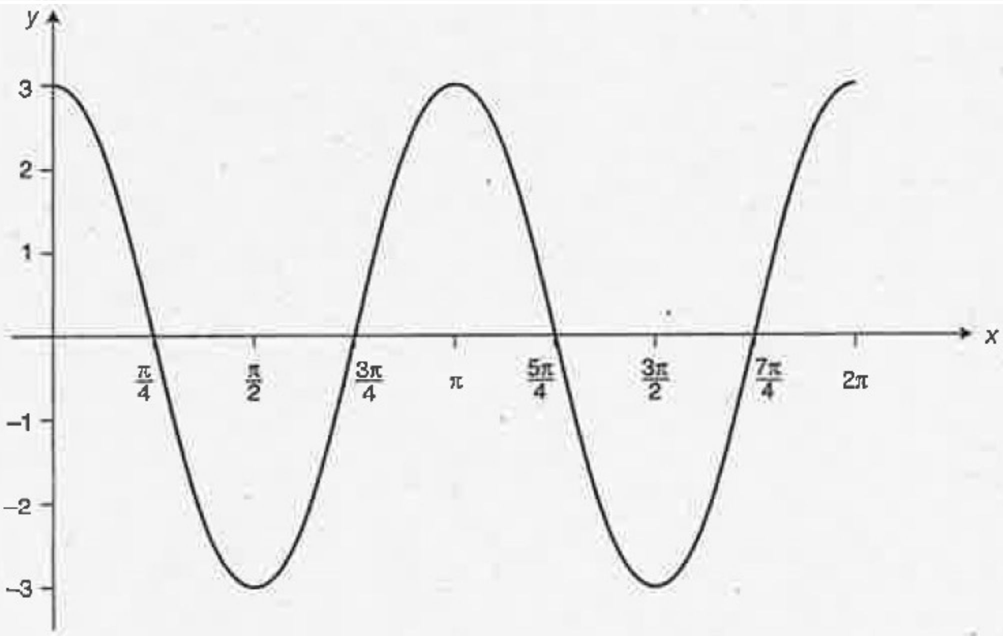
x-intercept, $\cos 2x = 0$, $2x \ \frac{n\pi}{2}$ (for n odd)

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Points to plot:

$(0, 3), \left(\frac{\pi}{4}, 0\right), \left(\frac{\pi}{2}, -3\right), \left(\frac{3\pi}{4}, 0\right), (\pi, 3), \left(\frac{5\pi}{4}, 0\right), \left(\frac{3\pi}{2}, -3\right), \left(\frac{7\pi}{4}, 0\right), (2\pi, 3)$



10. *(a)* $f(x) = ax^2 + 4x - 2$ discriminant $b^2 - 4ac$

For equal roots $b^2 - 4ac = 0$

$a = a,\ b = 4,\ c = -2$

$b^2 - 4ac = 16 - 4(a)(-2)$

$\Rightarrow \quad 16 + 8a = 0$

$8a = -16$

$a = -2$

y-intercept $(0, -2)$, x-intercept $(1, 0)$

$\frac{-b \pm \sqrt{0}}{2a}$ $a = -2$

$= \frac{-4}{-4} = 1$

$-2x^2$, shape 

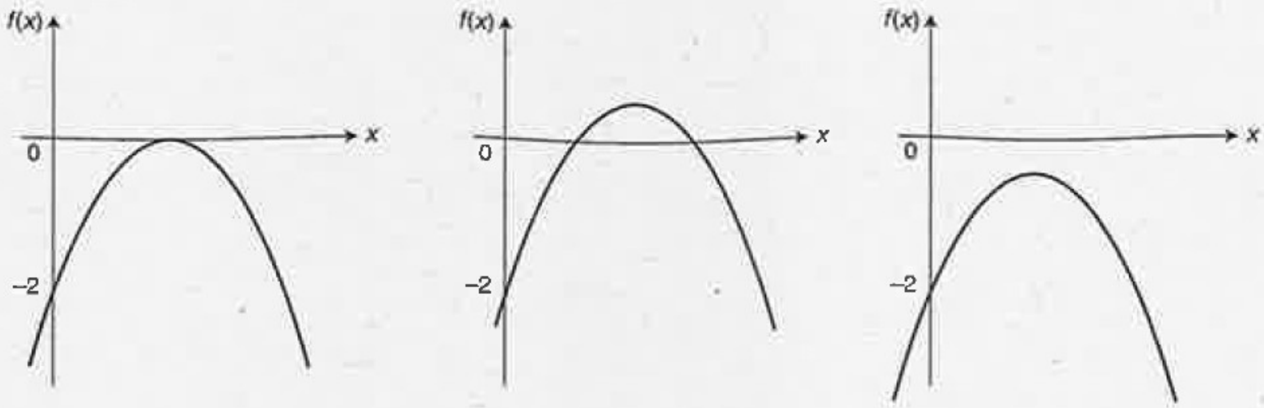
(b) $f(x)$ has real roots if $b^2 - 4ac > 0$

$$\Rightarrow 16 + 8a > 0$$
$$8a > -16$$
$$a > -2$$

(c) $f(x)$ has real roots if $b^2 - 4ac < 0$

$$\Rightarrow 16 + 8a < 0$$
$$8a < -16$$
$$a < -2$$

(d)



equal roots — real roots — no real roots

11. *(a)* Find the equation of the tangent to the curve $y = x^3 - 4x^2 + 2x$ at the point where $x = 1$.

The point of contact is $(1, f(1)) = (1, -1)$

$$f(x) = x^3 - 4x^2 + 2x$$
gradient = $f'(x) = 3x^2 - 8x + 2$

gradient = $f'(1) = 3(1)^2 - 8(1) + 2$

$$= -3$$
$$m = -3 \text{ through } (1, -1)$$
$$y - b = m(x - a)$$
$$y - (-1) = -3(x - 1)$$
$$y + 1 = -3x + 3$$
$y + 3x = 2$ is the required equation.

(b) $y + 3x = 2$; when $x = 0, y = 2$; when $y = 0, 3x = 2, x = \frac{2}{3}$

coordinates are $(0, 2)$ and $\left(\frac{2}{3}, 0\right)$

12. $A = (-1, 4, -2)$, $B = (1, 2, -3)$ and $C = (0, 3, -4)$,

$$\cos BAC = \frac{AB \cdot AC}{|AB||AC|}$$

$AB = \boldsymbol{b} - \boldsymbol{a}$

$AC = \boldsymbol{c} - \boldsymbol{a}$

$$\boldsymbol{b} - \boldsymbol{a} \qquad\qquad\qquad \boldsymbol{c} - \boldsymbol{a}$$

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} - \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} \qquad\qquad \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$BA \cdot BC = (2 \times 1) + (-2 \times -1) + (-1 \times -2) = 2 + 2 + 2 = 6$

$|AB|^2 = 2^2 + (-2)^2 + (-1)^2 = 9; \qquad |AB| = 3$

$|AC|^2 = 1^2 + (-1)^2 + (-2)^2 = 6; \qquad |AC| = \sqrt{6}$

$$\cos BAC = \frac{AB \cdot AC}{|AB||AC|} = \frac{6}{3 \times \sqrt{6}} = \frac{2}{\sqrt{6}} \text{ as given.}$$

Since the cosine of angle BAC is positive, the angle is in the first quadrant and is an acute angle.