Polynomials Past Papers Unit 2 Outcome 1

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Given $p(x) = x^2 + x - 6$, which of the following are true?

   I. $(x+3)$ is a factor of $p(x)$.

   II. $x = 2$ is a root of $p(x) = 0$.

   A. Neither I nor II is true

   B. Only I is true

   C. Only II is true

   D. Both I and II are true

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.1	C	0.78	0.67	NC	A21	HSN 170

$(x+3)$ is a factor $\Leftrightarrow p(-3) = 0$.

$p(-3) = (-3)^2 - 3 - 6 = 9 - 9 = 0$

So $(x+3)$ is a factor of $p(x)$.

$p(2) = 2^2 + 2 - 6 = 4 + 2 - 6 = 0$.

So $x = 2$ is a root of $p(x) = 0$. Option D

2. When $2ax^3 + (a+1)x - 6$ is divided by $x + 2$, the remainder is 2.

   What is the value of a?

   A. $\frac{5}{3}$

   B. $-\frac{4}{9}$

   C. $-\frac{5}{9}$

   D. $-\frac{5}{7}$

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	2.1	C	0.41	0.77	NC	A21	HSN 174

Let $f(x) = 2ax^3 + (a+1)x - 6$.

If the remainder is 2 then $f(-2) = 2$, ie

$$2a \times (-2)^3 + (a+1) \times (-2) - 6 = 2$$
$$-16a - 2a - 2 - 6 = 2$$
$$-18a = 10$$
$$a = -\frac{5}{9}$$

Option C

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA] 3. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x-a)^2 + b$. **2**

(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A5	$a = 2, b = 1$	2002 P1 Q7
(*b*)	4	C	NC	A3	sketch	
(*c*)	1	C	NC	A16, A6	$-1 < x < 5$	

•1 pd: process, e.g. completing the square •2 pd: process, e.g. completing the square •3 ic: interpret minimum •4 ic: interpret y-intercept •5 ss: reflect in x-axis •6 ss: translate parallel to y-axis •7 ic: interpret graph	•1 $a = 2$ •2 $b = 1$ •3 *any two from:* parabola; min. t.p. $(2,1)$; $(0,5)$ •4 the remaining one from above list •5 reflecting in x-axis •6 translating $+10$ units, parallel to y-axis •7 $(-1,5)$ i.e. $-1 < x < 5$

[SQA] 4. Find the values of x for which the function $f(x) = 2x^3 - 3x^2 - 36x$ is increasing. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	4	2.1	2	2					1.3.11	2.1.9	Source: 1996 P1 qu.16

•1	know to consider $f'(x) > 0$	stated or implied by the evidence for •4.
•2	$\frac{dy}{dx} = 6x^2 - 6x - 36$	
•3	$6(x-3)(x+2) > 0$	or by formula or completing the square
•4	$x < -2,\ x > 3$	

[SQA] 5.

(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots. **1**

(ii) Hence or otherwise show that the equation $x(x+1) = 3x - 2$ has no real roots. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(i)	1	2.1					1		2.1.6		Source
(ii)	2	2.1					2		2.1.6		1999 P1 qu.8

- •1 $b^2 - 4ac = 0$
- •2 $x^2 + 6x + 9 = 0$
- •3 $b^2 - 4ac = 36 - 36 = 0$ *OR* •3 $(x+3)(x+3) = 0$ so roots are $-3, -3$

[SQA] 6. Show that the roots of the equation $(k-2)x^2 - (3k-2)x + 2k = 0$ are real. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
	4	2.1					1	3	2.1.6		Source
											1990 P1 qu.18

- •1 *use* discriminant Δ
- •2 $\Delta = (3k-2)^2 - 8k(k-2)$
- •3 $\Delta = k^2 + 4k + 4$
- •4 $(k+2)^2 \geq 0$ so roots real

[SQA] 7. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A18	$k = \frac{1}{4}$	2001 P1 Q2

•1 ss: know to set disc. to zero •2 ic: substitute a, b and c into discriminant •3 pd: process equation in k	•1 $b^2 - 4ac = 0$ *stated or implied by* •2 •2 $(-5)^2 - 4 \times (k+6)$ •3 $k = \frac{1}{4}$

[SQA] 8. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers. 5

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	5	2.1	1	4					2.1.6		Source: 1991 P1 qu.18

- •1 for realising "$b^2 - 4ac$" ≥ 0
- •2 $kx^2 + 3x + (3-k) = 0$
- •3 $\Delta = 3^2 - 4k(3-k)$
- •4 $\Delta = (2k-3)^2$
- •5 for stating $(2k-3)^2$ is ≥ 0 for all real k

[SQA] 9. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	2	2.1	2						2.1.7		Source: 1993 P1 qu.3

- •1 discriminant $= 16 - 4 \times 2 \times k$
- •2 $16 - 8k \geq 0$ for real roots $\Rightarrow k \leq 2$

[SQA] 10. For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots? 2

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	2	2.1	2						2.1.7		Source: 1996 P1 qu.2

- •1 $b^2 - 4ac = 0$
- •2 $a = 2\frac{1}{2}$

[SQA] 11. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, 0.1	proof	2002 P2 Q9

•1 ss: know to use discriminant •2 ic: pick out discriminant •3 pd: simplify to quadratic •4 ss: choose to draw table or graph •5 pd: complete proof using disc.≥ 0	•1 discriminant $= \ldots$ •2 disc $= (-5k)^2 - 4(1-2k)(-2k)$ •3 $9k^2 + 8k$ •4 e.g. draw a table, graph, complete the square •5 complete proof and conclusion relating to disc.≥ 0

[SQA] 12. **The roots of the equation $(x-1)(x+k) = -4$ are equal.**
Find the values of k. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1 Source
.	5	2.1					1	4	2.1.7		1995 P1 qu.20

•1 $x^2 + kx - x + 4 - k = 0$
•2 $b^2 - 4ac = 0$
•3 $(k-1)^2 - 4(4-k)$
•4 $k^2 + 2k - 15 = 0$
•5 $k = -5,\ k = 3$

[SQA] 13. (*a*) $f(x) = 2x + 1, \quad g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(*a*)	4	1.2	4						1.2.6		Source **1996 Paper 2 Qu.4**
(*b*)	7	2.1	7						2.1.6, 2.1.7, 0.1		

(*a*)
- •1 $g(2x+1)$
- •2 $(2x+1)^2 + k$
- •3 $f\left(x^2 + k\right)$
- •4 $2\left(x^2 + k\right) + 1$

(*b*)
- •5 $4x^2 + 4x + k + 1$ *AND* $2x^2 + 2k + 1$
- •6 $4x^2 + 4x + k + 1 - \left(2x^2 + 2k + 1\right) = 0$ *so* $2x^2 + 4x - k = 0$
- •7 $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$
- •8 so roots real & distinct
- •9 $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$
- •10 $b^2 - 4ac = 0$ for equal roots
- •11 $k = -2$

[SQA] 14. Factorise fully $2x^3 + 5x^2 - 4x - 3$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	4	2.1	4						2.1.3		Source **1989 P1 qu.2**

- •1 *strat*: make 2 trial divisions or 2 trial evaluations
- •2 first linear factor
- •3 quadratic factor
- •4 other linear factors

$(x-1)(2x+1)(x+3)$

[SQA] 15. Find p if $(x+3)$ is a factor of $x^3 - x^2 + px + 15$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	3	2.1					3		2.1.1		Source: 1990 P1 qu.1

- •1 *strat*: e.g. find $f(-3)$
- •2 $f(-3) = 0$
- •3 $p = -7$

[SQA] 16. **(a) Show that $x = 2$ is a root of the equation $2x^3 + x^2 - 13x + 6 = 0$.** **1**
(b) Hence find the other roots. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	1	2.1	1						2.1.1		Source: 1999 P1 qu.1
(b)	3	2.1	3						2.1.2		

- •1 $f(2) = 16 + 4 - 26 + 6 = 0$
  or
  the appearance of a '0' at the end of the 3rd line in the table below
- •2

2	2	1	−13	6
		4	10	−6
	2	5	−3	0

- •3 $2x^2 + 5x - 3$
- •4 $-3, \frac{1}{2}$

[SQA] 17. Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	3	2.1					3		2.1.1		Source: 1992 P1 qu.3

- •1 $f(2) = 8 + 4k - 8 - 12$
- •2 $f(2) = 0$
- •3 $k = 3$

- •1 correct use of division
- •2 remainder $= 4k - 12$
- •3 $k = 3$

[SQA] 18. When $f(x) = 2x^4 - x^3 + px^2 + qx + 12$ is divided by $(x-2)$, the remainder is 114.
One factor of $f(x)$ is $(x+1)$.
Find the values of p and q. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1 Source
.	5	2.1					5		2.1.1		1991 P1 qu.6

- •1 $f(2) = 114$
- •2 $f(-1) = 0$
- •3 $4p + 2q = 78$
- •4 $p - q = -15$
- •5 $p = 8, q = 23$

[SQA] 19. One root of the equation $2x^3 - 3x^2 + px + 30 = 0$ is -3.
Find the value of p and the other roots. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1 Source
.	4	2.1	4						2.1.2		1993 P1 qu.7

- •1 $f(-3) = -54 - 27 - 3p + 30$ **or** synth. division e.g.

-3	2	-3	p	30
		-6	27	$-3p-81$
	2	-9	$p+27$	$-3p-51$

- •2 $p = -17$
- •3 $2x^2 - 9x + 10$ **and** $-3p - 51 = 0$
- •4 $2, \frac{5}{2}$

[SQA] 20. (a) Show that $(x-3)$ is a factor of $f(x)$ where $f(x) = 2x^3 + 3x^2 - 23x - 12$. **2**

(b) Hence express $f(x)$ in its fully factorised form. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1 Source
(a)	2	2.1	2						2.1.3		1995 P1 qu.2
(b)	2	2.1	2						2.1.3		

- •1 $f(3) = 2\times3^3 + 3\times3^2 - 23\times3 - 12$ | or equivalent division
- •2 $= 0$
- •3 $2x^2 + 9x + 4$
- •4 $(x-3)(2x+1)(x+4)$

[SQA] 21. Express $x^4 - x$ in its fully factorised form. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	4	2.1	4						2.1.3		Source: 1996 P1 qu.7

- $\bullet^1$ $x\left(x^3-1\right)$
- $\bullet^2$ synthetic division or eval. $f(k)$
- $\bullet^3$ linear factor $=(x-1)$
- $\bullet^4$ $x(x-1)\left(x^2+x+1\right)$

OR

- $\bullet^1$ synthetic division or eval. $f(k)$
- $\bullet^2$ linear factor $=(x-1)$
- $\bullet^3$ cubic factor $=\left(x^3+x^2+x\right)$
- $\bullet^4$ $x(x-1)\left(x^2+x+1\right)$

[SQA] 22. (*a*) Find a real root of the equation $2x^3 - 3x^2 + 2x - 8 = 0$. **2**

(*b*) Show algebraically that there are no other real roots. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	2	2.1	2						2.1.2		Source: 1997 P1 qu.5
(b)	3	2.1	3						2.1.7		

- $\bullet^1$ looking for $f(x) = \ldots\ldots = 0$
- $\bullet^2$ $x = 2$ **explicitly stated**
- $\bullet^3$ $2x^2 + x + 4$
- $\bullet^4$ $b^2 - 4ac = 1 - 4 \times 2 \times 4$
- $\bullet^5$ $b^2 - 4ac < 0$ means no real roots

[SQA] 23. Express $x^3 - 4x^2 - 7x + 10$ in its fully factorised form. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
.	4	2.1	4						2.1.3		Source: 1998 P1 qu.2

- $\bullet^1$ evaluating $f(k)$ for any integer by any method
- $\bullet^2$ find 1 value of k s.t. $f(k) = 0$
  e.g. $f(1)$ *or* $f(-2)$ *or* $f(5)$
- $\bullet^3$ quad factor *e.g.* $x^2 - 3x - 10$
- $\bullet^4$ $(x-1)(x+2)(x-5)$

[SQA] 24. (*a*) Given that $x+2$ is a factor of $2x^3+x^2+kx+2$, find the value of k. **3**

(*b*) Hence solve the equation $2x^3+x^2+kx+2=0$ when k takes this value. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	CN	A21	$k=-5$	2001 P2 Q1
(*b*)	2	C	CN	A22	$x=-2,\frac{1}{2},1$	

•1 ss: use synth division or f(evaluation) •2 pd: process •3 pd: process •4 ss: find a quadratic factor •5 pd: process	•1 $f(-2)=2(-2)^3+\cdots$ •2 $2(-2)^3+(-2)^2-2k+2$ •3 $k=-5$ •4 $2x^2-3x+1$ **or** $2x^2+3x-2$ **or** x^2+x-2 •5 $(2x-1)(x-1)$ **or** $(2x-1)(x+2)$ **or** $(x+2)(x-1)$ **and** $x=-2,\frac{1}{2},1$

[SQA] 25.

(*a*) Write the equation $\cos 2\theta+8\cos\theta+9=0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(*b*) Show that there are no real roots for θ. **1**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(a)	3	2.3					1	2	2.3.3	2.1.6	Source
(b)	1	1.2						1	1.2.1		**1998 P1 qu.18**

•1 $2\cos^2\theta-1+8\cos\theta+9$

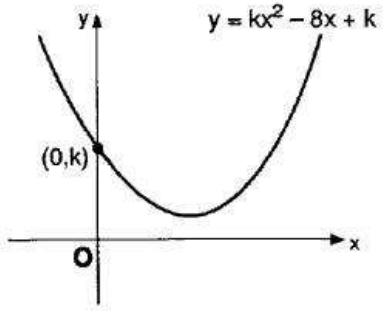
•2 $2(\cos\theta+2)^2=0$

or "b^2-4ac" $=16-4\times1\times4$

•3 $\cos\theta=-2$ twice or "b^2-4ac" $=0$

•4 $\cos\theta=-2$ has no solution

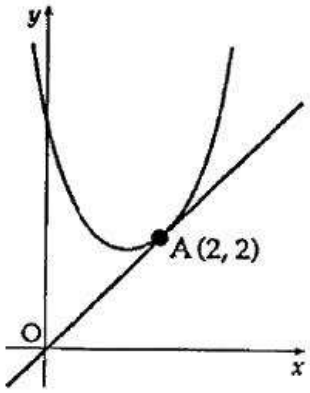
[SQA] 26. **Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis.** 4



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.1
.	4	2.1	1	3					2.1.7	Source 1992 P1 qu.17

- •1 *strat*: use discriminant
- •2 $b^2 - 4ac < 0$
- •3 $64 - 4k^2$
- •4 $k = 5$

[SQA] 27. (a) The point A(2, 2) lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q. (1)



(b) The tangent to the parabola at A is the line $y = x$. Find the value of p.
Hence find the equation of the parabola. (6)

(c) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value? (2)

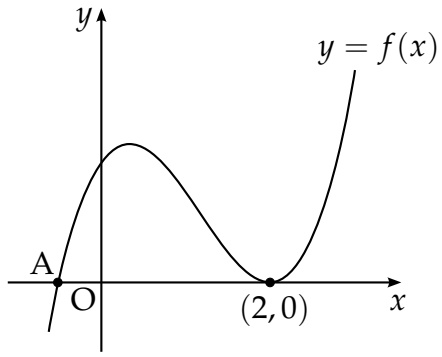
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	1	0.1					1		0.1		Source
(b)	6	1.3					2	4	1.3.7,	0.1	1994 Paper 2
(c)	2	2.1						2	2.1.6		Qu.9

(a) •1 $2p + q = -2$

(b) •2 strategy
•3 $2x + p$
•4 gradient = 1, or equivalent
•5 $4 + p$
•6 $p = -3$
•7 $q = 4$

(c) •8 $\Delta = -7$
•9 $\sqrt{-7}$ means no roots

[SQA] 28. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

$\bullet^1$ ss: know to differentiate
$\bullet^2$ pd: differentiate
$\bullet^3$ ss: know to set derivative to zero
$\bullet^4$ pd: start solving process of equation
$\bullet^5$ pd: complete solving process

$\bullet^6$ ss: strategy for cubic, e.g. synth. division
$\bullet^7$ ic: extract quadratic factor
$\bullet^8$ pd: complete the cubic factorisation

$\bullet^9$ ic: interpret the factors
$\bullet^{10}$ ic: interpret the diagram

$\bullet^1$ $f'(x) = \ldots$
$\bullet^2$ $6x^2 - 14x + 4$
$\bullet^3$ $6x^2 - 14x + 4 = 0$
$\bullet^4$ $(3x-1)(x-2)$
$\bullet^5$ $x = \frac{1}{3}$

$\bullet^6$

$\ldots$	2	-7	4	4
		$\ldots$	$\ldots$	$\ldots$
	$\ldots$	$\ldots$	$\ldots$	0

$\bullet^7$ $2x^2 - 3x - 2$
$\bullet^8$ $(x-2)(2x+1)(x-2)$

$\bullet^9$ $A(-\frac{1}{2}, 0)$
$\bullet^{10}$ $x < -\frac{1}{2}$

[SQA] 29.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.
The function g is defined by $g(x) = x - 1$.
Show that $f\big(g(x)\big) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f\big(g(x)\big)$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f\big(g(x)\big)}$.
For what values of x is the function k not defined? **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		2.1
			C	A/B	C	A/B	C	A/B	Main	Additional	
(*a*)	4	1.2	4						1.2.6		Source
(*b*)	3	2.1	3						2.1.3		1990 Paper 2
(*c*)	2	1.2	2						1.2.1		Qu. 6

(*a*) •1 $f(g(x)) = f(x-1)$
•2 $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$
•3 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$
•4 $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

(*b*) •5 *first "0" e.g.*

2	1	−5	2	8
		2	−6	−8
	1	−3	−4	0

•6 $x^2 - 3x - 4 = (x+1)(x-4)$
•7 $(x-2)(x+1)(x-4)$

(*c*) •8 denominator $(= (x-2)(x+1)(x-4)) \neq 0$
•9 $-1,\ 2,\ 4$

[SQA] 30. The graph of the curve with equation $y = 2x^3 + x^2 - 13x + a$ crosses the x-axis at the point (2,0).

(a) Find the value of a and hence write down the coordinates of the point at which this curve crosses the y-axis. **(3)**

(b) Find algebraically the coordinates of the other points at which the curve crosses the x-axis. **(4)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	3	2.1	3						2.1.3		Source **1994 Paper 2 Qu.1**
(b)	4	2.1	4						2.1.3		

(a) $\bullet^1$ strategy

eg

	2	1	−13	a
2		4	10	−6
	2	5	−3	0

or $f(2) = 0 = 16 + 4 - 26 + a$

$\bullet^2$ $a = 6$

$\bullet^3$ $(0, 6)$

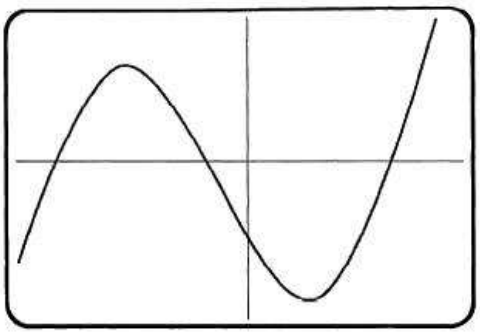
(b) $\bullet^4$ $2x^2 + 5x - 3$

$\bullet^5$ $(x + 3)(2x - 1)$

$\bullet^6$ $x = -3, \frac{1}{2}$

$\bullet^7$ $(-3, 0), \left(\frac{1}{2}, 0\right)$

[SQA] 31. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(a) Factorise $f(x)$. (3)

(b) Write down the co-ordinates of the four points where the curve crosses the x and y axes. (2)

(c) Find the turning points and justify their nature. (6)

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	3	2.1	3						2.1.3		Source
(b)	2	1.2	2						1.2.9		1992 Paper 2
(c)	6	1.3	6						1.3.12		Qu.1

(a) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(b) •4 For all 3 points on x-axis

•5 $(0,-16)$

(c) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, **and** $x = -\frac{8}{3}$

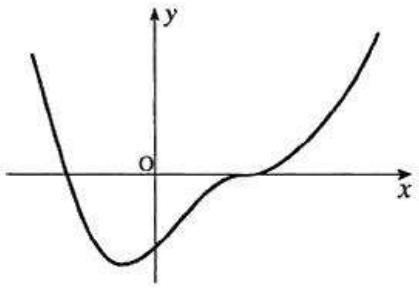
•9 $y = -36$, **and** $y = \frac{400}{27}(14.8)$

•10

	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	+	0	−	−	0	+
	⋰	…	⋱	⋱	…	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2,-36)$

[SQA] 32. The function f, whose incomplete graph is shown in the diagram, is defined by $f(x) = x^4 - 2x^3 + 2x - 1$.
Find the coordinates of the stationary points and justify their nature. **(8)**



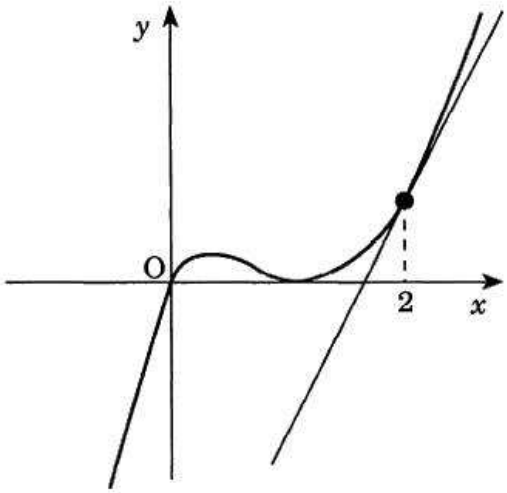
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
-	8	2.1					8		2.1.3,	1.3.12	Source 1993 Paper 2 Qu.1

- •1 for knowing to differentiate
- •2 $f'(x) = 4x^3 - 6x^2 + 2$
- •3 for putting $f'(x) = 0$
- •4 for factorising **or** checking zeros
- •5 $x = -\frac{1}{2}$, $x = 1$
- •6 $y = -\frac{27}{16}$, $y = 0$
- •7 completed nature table

x	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	< 1	1	> 1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	\	—	/	/	—	/

- •8 $(1,0)$ is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

[SQA] 33. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	4	1.3	4						1.3.9,	1.1.7	Source
(b)	5	2.1	5						2.1.2,	2.1.8	**1995 Paper 2 Qu.2**

(a) •1 $\frac{dy}{dx} = \ldots\ldots$

•2 $3x^2 - 4x + 1$

•3 $m_{x=2} = 5$

•4 $y - 2 = 5(x - 2)$

(b) •5 equate 'y's

•6 $x^3 - 2x^2 - 4x + 8 = 0$

•7 e.g. synthetic division

•8 the appearance of:

$x^2 - 4$

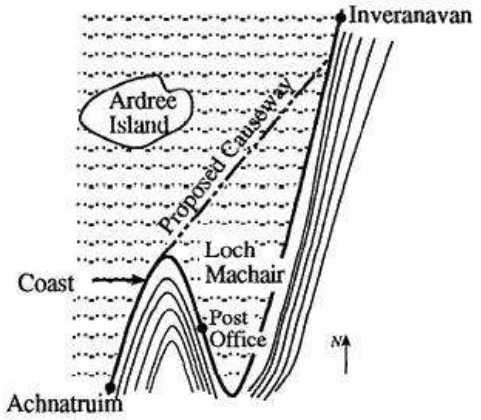
or $x^2 - 4x + 4$

or ± 2

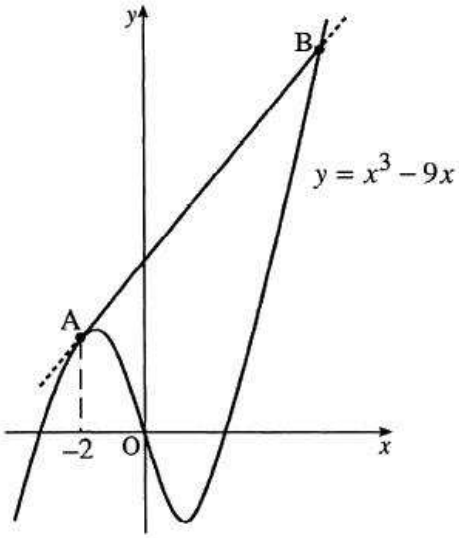
or $-2, 2, 2$

•9 $x = -2,\ y = -18$

[SQA] 34. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :	**2.1**
			C	A/B	C	A/B	C	A/B	Main Additional	
(*a*)i	1	0.1	1						0.1	Source
(*a*)ii	4	1.1	4						1.1.6, 4	**1998 Paper 2**
(*b*)	7	2.1	2	5					2.1.12 & 2.1.2	**Qu. 5**

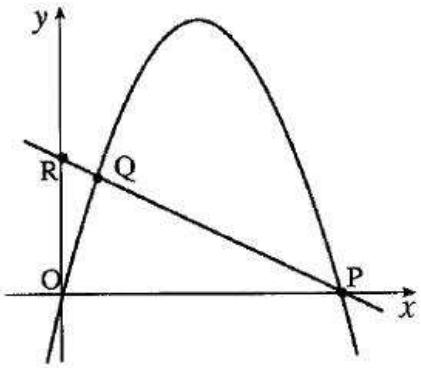
(*a*)
- •1 $y_{x=-2} = 10$
- •2 $\frac{dy}{dx} = \ldots\ldots$
- •3 $3x^2 - 9$
- •4 $m_{x=-2} = 3$
- •5 $y - 10 - 3(x+2)$

(*b*)
- •6 $y = 3x + 16$
- •7 $3x + 16 = x^3 - 9x$
- •8 $x^3 - 12x - 16 = 0$
- •9 *e.g.*

	1	0	–12	–16
–2		–2	4	16
	1	–2	–8	0

- •10 *e.g.* $x^2 - 2x - 8$
- •11 *e.g.* $(x+2)(x-4)$
- •12 B is $(4, 28)$

[SQA] 35. The parabola shown in the diagram has equation $y = 4x - x^2$ and intersects the x-axis at the origin and P.



(a) Find the coordinates of the point P. 2

(b) R is the point (0, 2). Find the equation of PR. 2

(c) The line and the parabola also intersect at Q. Find the coordinates of Q. 4

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.1
(a)	2	1.2	2						1.2.9		Source
(b)	2	1.1	2						1.1.7		1999 Paper 2
(c)	4	2.1	4						2.1.8		Qu. 4

(a) •1 $4x - x^2 = 0$ *stated or implied by* •2

•2 $(4, 0)$

(b) •3 $m = -\frac{1}{2}$

•4 $y = -\frac{1}{2}x + 2$

or $y - 2 = -\frac{1}{2}(x - 0)$

or $y - 0 = -\frac{1}{2}(x - 4)$

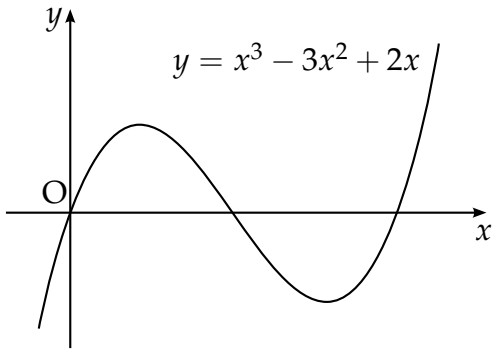
(c) •5 $4x - x^2 = 2 - \frac{1}{2}x$

•6 *e.g.* $2x^2 - 9x + 4 = 0$

•7 $x = \frac{1}{2}, x = 4$

•8 Q is $\left(\frac{1}{2}, \frac{7}{4}\right)$

[SQA] 36. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

$\bullet^1$ ss: know to differentiate
$\bullet^2$ pd: differentiate correctly
$\bullet^3$ ss: know that gradient $= f'(1)$
$\bullet^4$ ss: know that y-coord $= f(1)$
$\bullet^5$ ic: state equ. of line

$\bullet^6$ ss: equate equations
$\bullet^7$ pd: arrange in standard form
$\bullet^8$ ss: know how to solve cubic
$\bullet^9$ pd: process
$\bullet^{10}$ ic: interpret

$\bullet^1$ $y' = \ldots$
$\bullet^2$ $3x^2 - 6x + 2$
$\bullet^3$ $y'(1) = -1$
$\bullet^4$ $y(1) = 0$
$\bullet^5$ $y - 0 = -1(x - 1)$

$\bullet^6$ $2x - 4 = x^3 - 3x^2 + 2x$
$\bullet^7$ $x^3 - 3x^2 + 4 = 0$
$\bullet^8$

$\ldots$	1	-3	0	4
		$\ldots$	$\ldots$	$\ldots$
	$\ldots$	$\ldots$	$\ldots$	$\ldots$

$\bullet^9$ identify $x = -1$ from working
$\bullet^{10}$ $(-1, -6)$

[END OF WRITTEN QUESTIONS]