Quadratic Theory

1. Write each of the following quadratic expressions in the form $a(x+b)^2 + c$:

(a) $x^2 + 6x - 3$ (b) $x^2 - 5x + 1$ (c) $3x^2 + 12x + 2$

(d) $2x^2 - 6x - 4$ (e) $4 + 8x - x^2$ (f) $1 - 4x - 2x^2$

2. (a) Show that the function $f(x) = 2x^2 - 16x + 7$ can be written in the form $f(x) = a(x+p)^2 + q$ and write down the values of a, p and q.

Hence state the minimum value of the function and the corresponding value of x.

(b) Express the function $g(\alpha) = \sin^2\alpha - \sin\alpha - 1$ in the form $g(\alpha) = (\sin\alpha + p)^2 + q$ and write down the values of p and q.

Given that $0 < \alpha < \pi/2$, state the minimum value of g and the corresponding replacement for α.

3. (a) Find the value of k which results in the equation $kx^2 + 2kx - 1 = 0$ having **equal** roots, given that $k \neq 0$?

(b) A quadratic equation is given as $x^2 + (p-3)x + (\frac{1}{4} - 3p) = 0$.

For what values of p will the above equation have i) equal roots ;
ii) no real roots.

(c) Show that the roots of the equation $(t-1)x^2 + 2tx + 4 = 0$ are real for all values of t.

4. A computer generating random products using two variables delivers out the following four expressions on printed cards :

card 1 $4x^2$

card 2 $2 - k$

card 3 $2k + 1$

card 4 $4x + 1$

(a) Given that the product of cards 1 and 3 is equal to the product of cards 2 and 4, show that the following equation can be constructed

$$(8k+4)x^2 + (4k-8)x + (k-2) = 0$$

(b) Hence find the values of k so that the above equation has equal roots.