Recurrence Relations Homework

1. A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

(a) Determine the values of u_1 and u_2. **1**

(b) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q. **3**

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does this other sequence not have a limit? **3**

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$$u_{n+1} = au_n + b,$$

where a and b are constants and n is the number of years since the reserve was set up.

(a) Use the information above to find the values of a and b. **4**

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer. **3**

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

4 A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of u_4. 1

(b) Explain why this sequence approaches a limit as $n \to \infty$. 1

(c) Calculate this limit. 2

5 A sequence is generated by the recurrence relation $u_{n+1} = m u_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. 2

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. 1

(ii) Calculate this limit. 2

6 A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. 2

(*b*) Find the limit of this recurrence relation as $n \to \infty$. 2

7 (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4. 2

(*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

(i) Express u_1 and u_2 in terms of m.

(ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. 5