Sequences and Recurrence Relations 2

1. A sequence of numbers is defined by the recurrence relation $U_{n+1} = kU_n + c$, where k and c are constants.

   (a) Given that $U_0 = 10$, $U_1 = 14$ and $U_2 = 17 \cdot 2$, find **algebraically**, the values of k and c.

   (b) Calculate the value of E given that $E = \frac{3}{4}(L - 2)$, where L is the limit of this sequence.

2. A sequence is defined by the recurrence relation $U_{n+1} = 0 \cdot 8U_n + 3$.

   (a) Explain why this sequence has a limit as $n \to \infty$.

   (b) Find the limit of this sequence.

   (c) Taking $U_0 = 10$ and L as the limit of the sequence, find n such that

   $$L - U_n = 2 \cdot 56$$

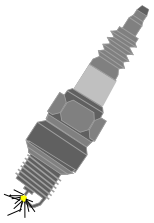
3. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$, where a and b are constants.

   (*a*) Given that $U_0 = 4$ and $b = -8$, express U_2 in terms of a.

   (*b*) Hence find the value of a when $U_2 = 88$ and $a > 0$.

   (c) Given that $S_3 = U_1 + U_2 + U_3$, calculate the value of S_3.

4. Over a period of time the effectiveness of a standard spark plug slowly decreases. It has been found that, in general, a spark plug will loose 8% of its burn efficiency every **two months** while in average use.

   (a) A new spark plug is allocated a *Burn Efficiency Rating (BER)* of 120 units.
   What would the *BER* be for this plug after a year of average use ?

   Give your answer correct to one decimal place.

   (b) After exhaustive research, a new fuel additive was developed.
   This additive , when used at the end of every **four month** period, immediately allows the *BER* to increase by 8 units.

   A plug which falls below a *BER* of 80 units should immediately be replaced.

   What <u>should</u> be the maximum recommended lifespan for a plug , in months, when using this additive ?