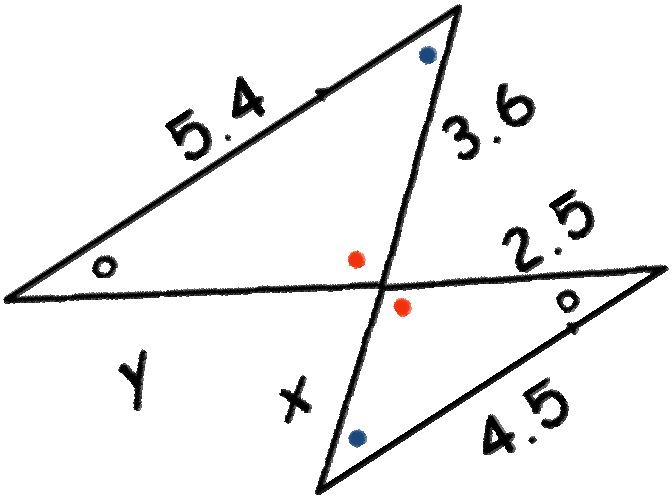
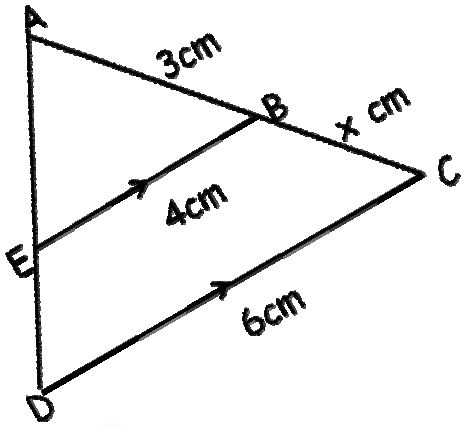
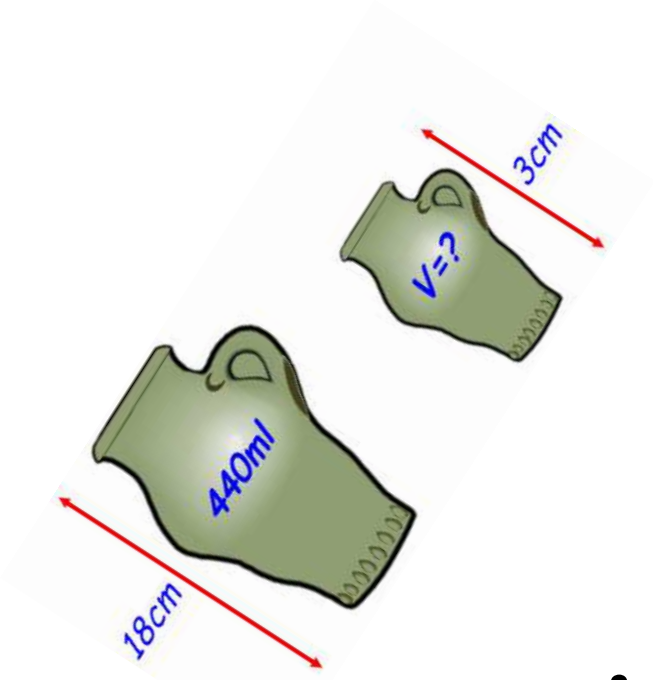
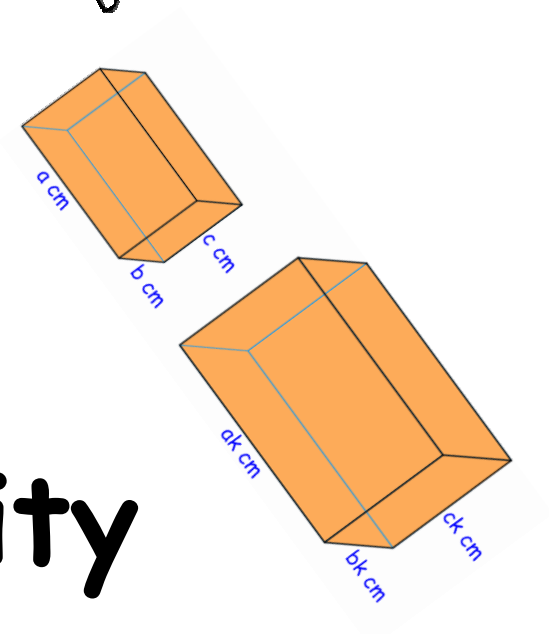
S4 National 5 Notes Chapter 3









Similarity

SIMILARITY

Representative Fractions

A scale can be give as a ratio e.g. on a map 1:250000. This is sometimes called the representative fraction.

In the above example, 1 unit on the map stands for 250000 units on the ground. This unit can be in centimetres, inches, metres etc. As the representative fraction works for all units.

Ex1

What length will represent a distance of 4km on a map with a scale of 1:50000?

$$4\text{km} = 4000\text{m} = 400{,}000\text{cm}$$

$$\text{Distance on map} = 400{,}000 \div 50{,}000$$
$$= 8\text{cm}$$

Similarly:

$$4 \div 50000 = 0.00008 \text{ km}$$
$$0.00008 \text{ km} = 0.08\text{m}$$
$$0.08 \text{ m} = 8\text{cm}$$

In order to work out a representative fraction we need to know one distance on the diagram and the actual distance.

Ex2

The map distance from Beinn Tarsuinn and Beinn Bhreac is 2 cm.
The actual distance is 5 km.
Find the representative fraction.

2cm : 5km
2cm : 500000cm
1cm : 250000cm
1 : 250000

Similar Triangles

If two shapes are exactly the same shape and size they are said to be congruent.

If two shapes are exactly the same shape but different sizes they are said to be similar.

If two triangles are similar and we know which angles correspond we can work out the missing sides.

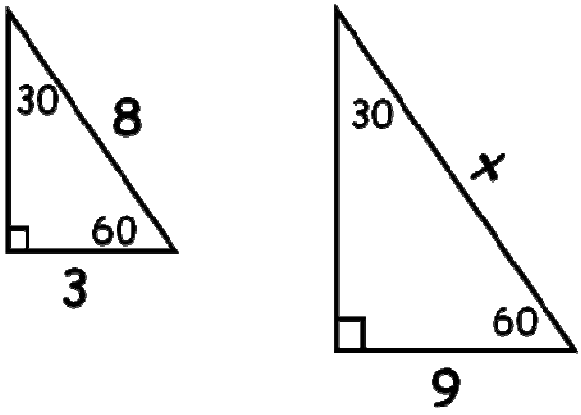
Ex4

These two triangles are similar. Find the length of x.

Solution

Notice that the side marked x is an enlargement of the side marked 8.

And that the side marked 9 is an enlargement of the side marked 3.



$$\text{Enlargement Scale Factor} = \frac{\text{Measurement From Larger}}{\text{Measurement From Smaller}}$$

$$\text{Es.f} = \frac{9}{3} = 3$$ i.e. the large triangle is 3 times bigger than the smaller.

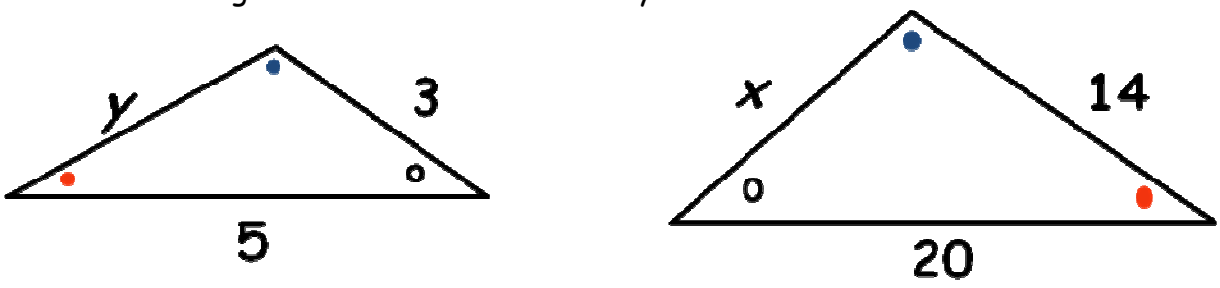
So x = Scale Factor $\times$ 8

$x = 3 \times 8$

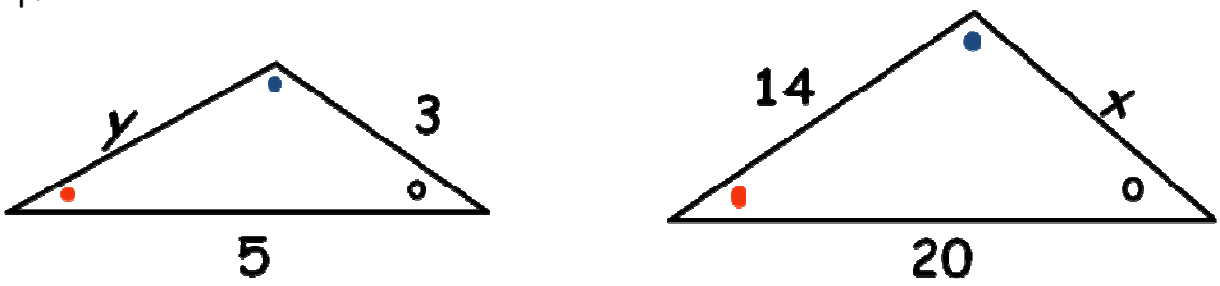
$x = 24cm$

Ex5

These two triangles are similar. Find x and y.



Draw the triangles so that the corresponding triangles match up.



x is an enlargement of the side marked 3 so:

$$\text{Es.f} = \frac{20}{5} = 4$$

$$\begin{aligned} \text{So } x &= \text{Scale Factor} \times 3 \\ &= 4 \times 3 \\ x &= 12 \end{aligned}$$

y is a reduction of the side marked 14 so:

$$\text{Reduction Scale Factor} = \frac{\text{Measurement From Smaller}}{\text{Measurement From Larger}}$$

$$\text{Rs.f} = \frac{5}{20} = \frac{1}{4} = 0.25$$

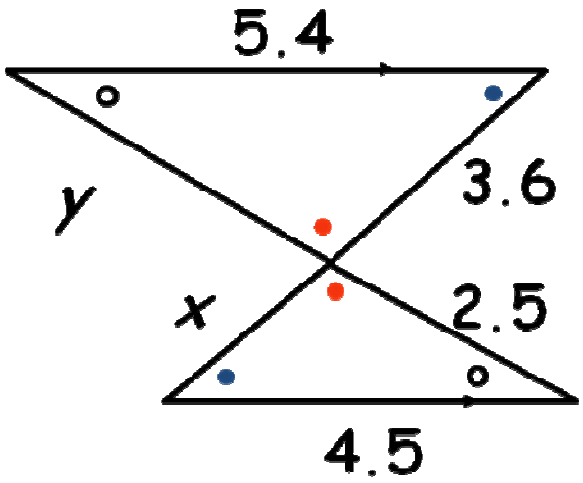
$$\begin{aligned} \text{So } y &= \text{Scale Factor} \times 14 \\ &= 0.25 \times 14 \\ y &= 3.5 \end{aligned}$$

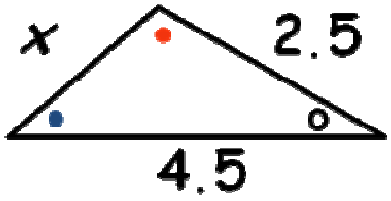
Similar Triangles – Harder Examples

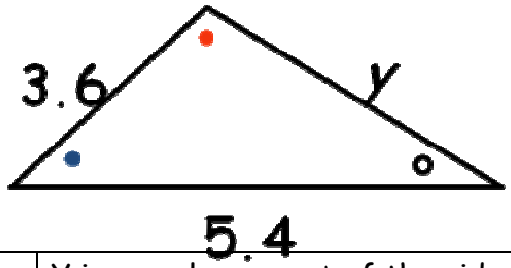
Ex6

These two triangles are similar. Find x and y.

Start by drawing the triangles out separately so that the corresponding angles match.





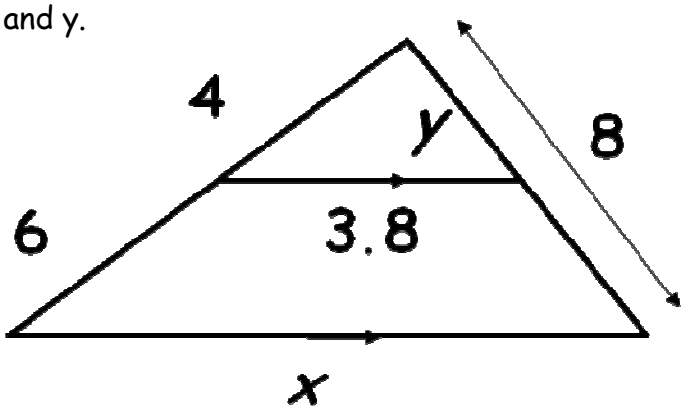


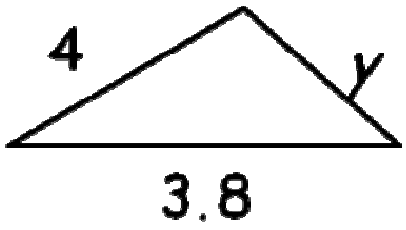
x is a reduction of the side marked 3.6 so:	Y is an enlargement of the side marked 2.5 so:
$\text{Rs.f} = \frac{4.5}{5.4} = \frac{5}{6}$	$\text{Es.f} = \frac{5.4}{4.5} = \frac{6}{5} = 1.2$
$\text{So } x = \text{Scale Factor} \times 3.6$ $= 0.83 \times 3.6$ $x = 3cm$	$\text{So } y = \text{Scale Factor} \times 2.5$ $= 1.2 \times 2.5$ $y = 3cm$

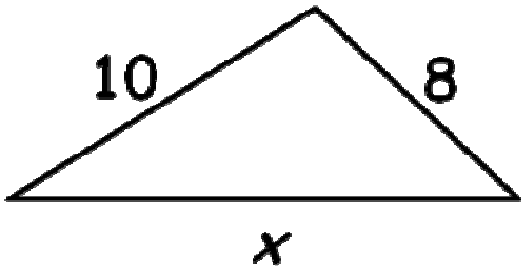
Ex7

These two triangles are similar. Find x and y.

Again, start by drawing the triangles out separately so that the corresponding angles match.



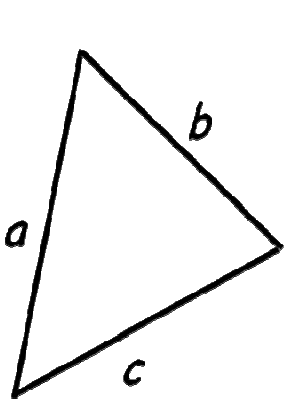


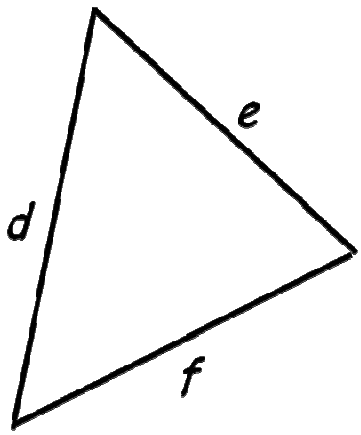


x is an enlargement of the side marked 3.8 so: $\text{Es.f} = \frac{10}{4} = \frac{5}{2} = 2.5$ $\text{So } x = \text{Scale Factor} \times 3.8$ $= 2.5 \times 3.8$ $x = 9.5cm$	y is a reduction of the side marked 8 so: $\text{Rs.f} = \frac{4}{10} = \frac{2}{5} = 0.4$ $\text{So } y = \text{Scale Factor} \times 8$ $= 0.4 \times 8$ $y = 3.2cm$

More Similar Triangles

In a pair of similar triangles, the ratio of corresponding sides are equal:



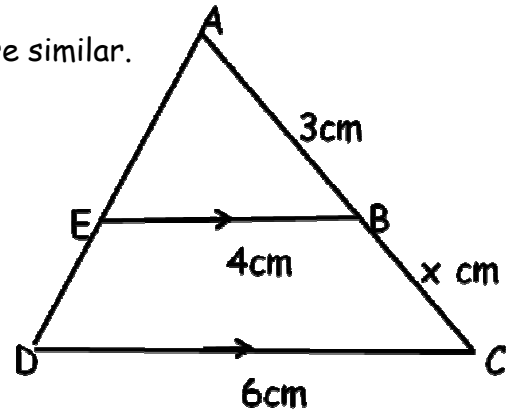


$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

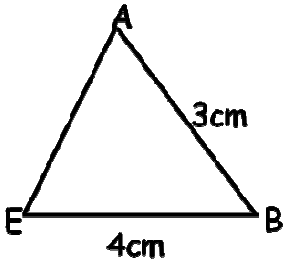
We say the corresponding sides are in proportion. This can be really useful when solving problems.

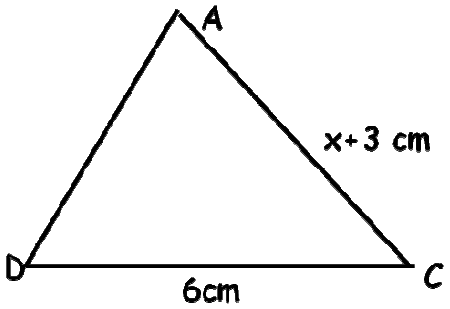
Ex8

Find the value of x given that the triangles are similar.



Start by drawing out both triangles so that the corresponding angles match up.





$$\frac{AB}{AC} = \frac{EB}{DC}$$

$$\frac{3}{x+3} = \frac{4}{6}$$

$$3 \times 6 = 4(x+3)$$

$$18 = 4x + 12$$

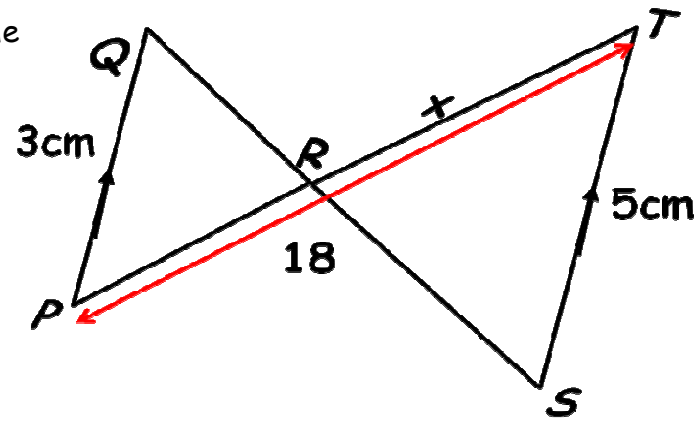
$$18 - 12 = 4x$$

$$6 = 4x$$

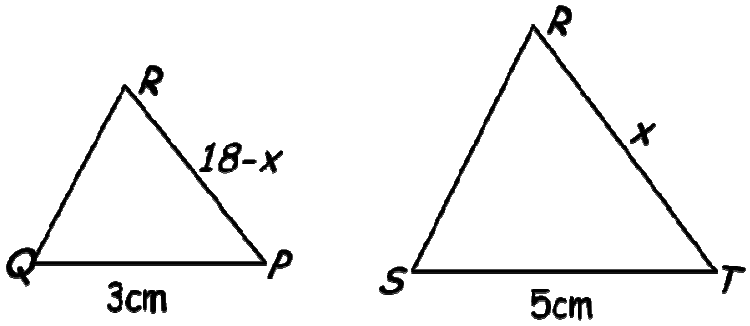
$$x = 1.5cm$$

Ex9

Find the value of x given that the triangles are similar.



Start by drawing out both triangles so that the corresponding angles match up.



$$\frac{PR}{RT} = \frac{QP}{ST}$$

$$\frac{18-x}{x} = \frac{3}{5}$$

$$5(18-x) = 3x$$

$$90 - 5x = 3x$$

$$90 = 3x + 5x$$

$$90 = 8x$$

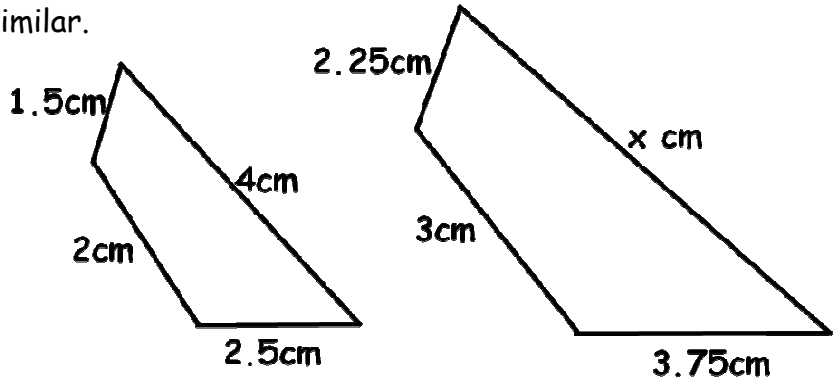
$$x = 11.25cm$$

Other Similar Figures

Any shapes that are similar, like triangles, will have their corresponding sides constant.

Ex10

These quadrilaterals are similar.
(a) Find the scale factor
(b) Find the value of x



Start by finding the scale factor:
x is an enlargement of the side marked 4cm so:

$\text{Es.f} = \frac{3}{2} = 1.5$ Note that we could have used any of the corresponding sides:

$$\frac{3}{2} = \frac{2.25}{1.5} = \frac{3.75}{2.5} = 1.5$$

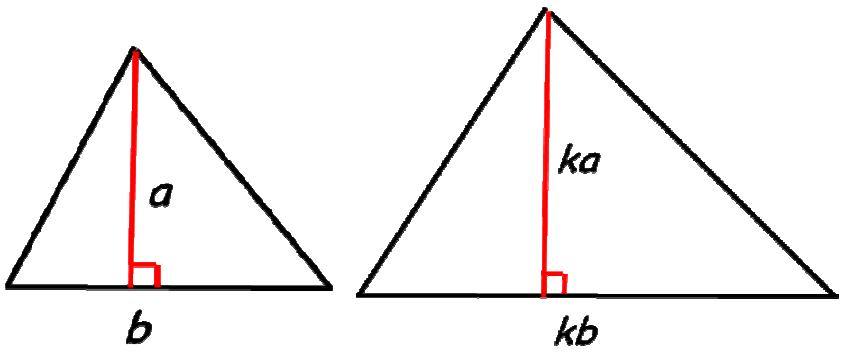
Then find the value of x:

$x = \text{Scale Factor} \times 4$
$x = 1.5 \times 4$
$x = 6cm$

Ratio of Areas

These two triangles are similar.

The enlargement factor is k.



Area of smaller triangle $= \frac{1}{2}ab$

Area of larger triangle $= \frac{1}{2} \times ka \times kb = \frac{1}{2}abk^2$

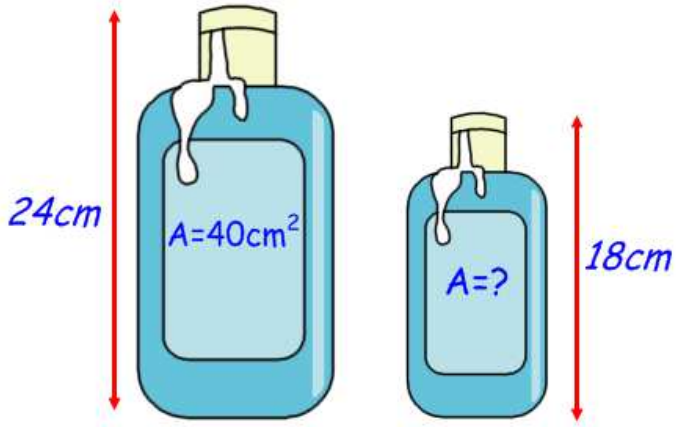
The larger area is found by multiplying the smaller by k^2

The scale factor for area is the scale factor for the length squared

Ex11

Two sun cream bottles are similar with heights as shown.

The larger bottle has a label of area $40cm^2$.



What is the area of the label on the smaller bottle?

Reduction factor (height) $= 18 \div 24 = 0.75$

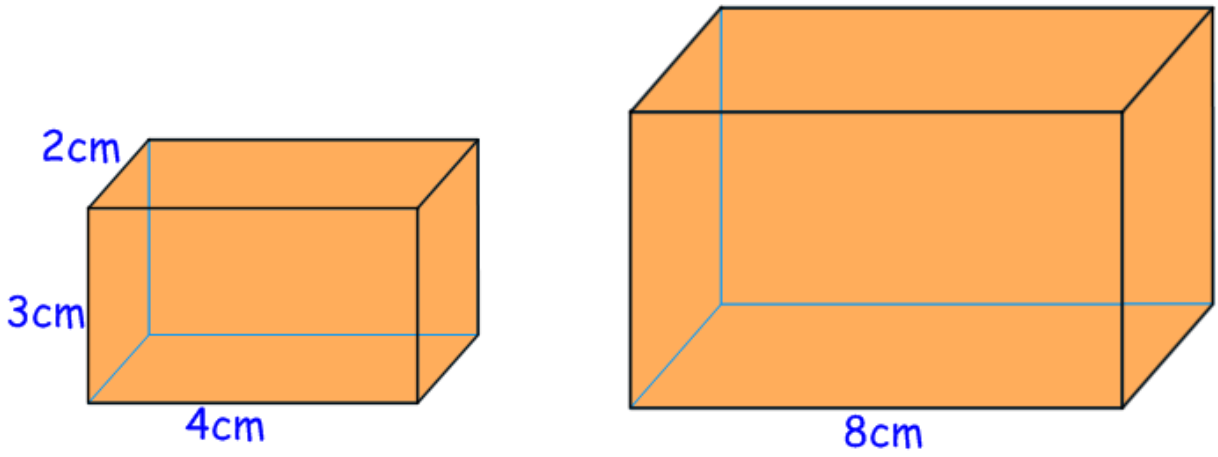
Reduction factor for area $= 0.75^2 = 0.5625$

Area of smaller label $= 40 \times 0.5625 = 22.5cm^2$

Note that it doesn't matter what shape the labels are, as long as they are similar.

Ex12

These two cuboids are similar. Calculate the surface area of the larger cuboid.



$$\text{S.A of smaller cuboid} = 2\times(2\times3)+2\times(2\times4)+2\times(3\times4)$$

$$= 52cm^2$$

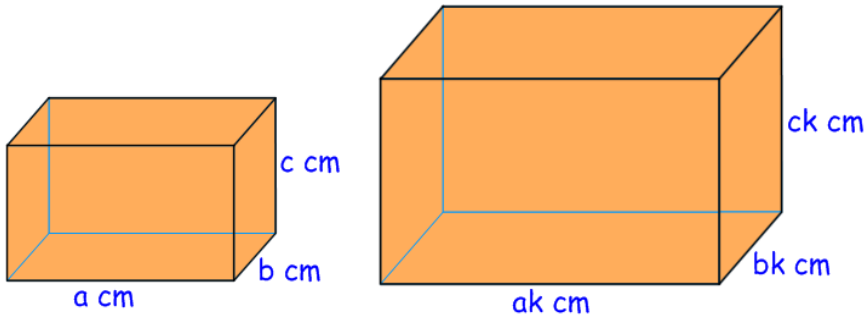
$$\text{Enlargement factor (length)} = 8\div4 = 2$$

$$\text{Enlargement factor for area} = 2^2 = 4$$

$$\text{S.A of larger cuboid} = 4\times52 = 208cm^2$$

Volume of Similar Solids

Two cuboids are similar (with an enlargement factor of k.)



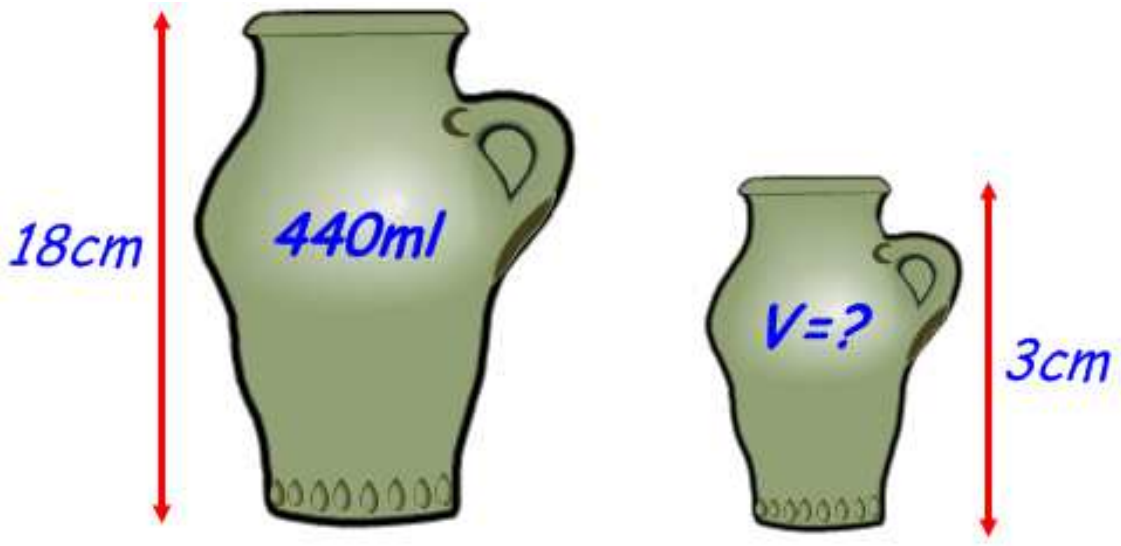
$$\text{Volume of smaller cuboid} = abc \text{ cm}^3$$

$$\text{Volume of larger cuboid} = akbkck = abck^3 \text{ cm}^3$$

The scale factor for volume is the scale factor for the length cubed

Ex13

A jug of milk has a height of 18cm holds 440ml of liquid. A smaller jug of milk in a similar shaped bottle has a height of 3cm. What volume of milk is in the smaller jug?



$$\text{Reduction factor (height)} = \frac{3}{18} = \frac{1}{6}$$

$$\text{Reduction factor (volume)} = \left(\frac{1}{6}\right)^3 = \frac{1}{216}$$

$$\text{Volume of the sample} = \frac{1}{216} \times 440 = 2.04\text{ml}$$