Trig. Past Papers Unit 2 Outcome 3

Written Questions

[SQA] 1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \le x \le 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	$60, 131{\cdot}8, 228{\cdot}2, 300$	2000 P2 Q5

•1 ss: know to use $\cos 2x = 2\cos^2 x - 1$	•1 $3(2\cos^2 x^\circ - 1)$
•2 pd: process	•2 $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$
•3 ss: know to/and factorise quadratic	•3 $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$
•4 pd: process	•4 $\cos x^\circ = \frac{1}{2}$, $x = 60, 30$
•5 pd: process	•5 $\cos x^\circ = -\frac{2}{3}$, $x = 132, 228$

[SQA] 2. Solve the equation $\cos 2x^\circ + 5\cos x^\circ - 2 = 0$, $0 \le x < 360$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3 Source
.	5	2.3			1	4			2.3.5		1994 P1 qu.15

•1 Replacing $\cos 2x$ by $2\cos^2 x - 1$	•5 300° and no extraneous solutions and no solution for $\cos x = -3$ indicated. [If a reason is given, it must be valid].
•2 $2\cos^2 x + 5\cos x - 3 = 0$	
•3 $(2\cos x - 1)(\cos x + 3) = 0$	
•4 60°	

[SQA] 3. Find the exact solutions of the equation $4\sin^2 x = 1$, $0 \le x < 2\pi$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3 Source
.	4	2.3	4						2.3.1	1.2.1	1995 P1 qu.8

•1 know to factorise, take square roots	•1 replace $\sin^2 x$ by $\frac{1}{2}(1 - \cos 2x)$
•2 $\sin x = \frac{1}{2}$	•2 $\cos 2x = \frac{1}{2}$
•3 $x = \frac{\pi}{6}, \frac{5\pi}{6}$	•3 $2x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$
•4 $\sin x = -\frac{1}{2}$ **and** $x = \frac{7\pi}{6}, \frac{11\pi}{6}$	•4 $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

[SQA] 4. Solve the equation $2\cos^2 x = \frac{1}{2}$, for $0 \leq x \leq \pi$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	3	2.3	3						2.3.1	1.2.1	Source **1990 P1 qu.15**

•[1] $\cos x = \pm\frac{1}{2}$
•[2] $x = \frac{\pi}{3}$
•[3] $\frac{2\pi}{3}$

[SQA] 5. Solve the equation $\cos 2x° + \cos x° = 0$, $0 \leq x < 360$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	5	2.3				5			2.3.5		Source **1995 P1 qu.15**

•[1] substitute $2\cos^2 x° - 1$ for $\cos 2x°$
•[2] $(2\cos x° - 1)(\cos x° + 1) = 0$
•[3] $\cos x° = \frac{1}{2}$, $\cos x° = -1$
•[4] $x = 60, 300$
•[5] $x = 180$

[SQA] 6. Solve $2\sin 3x° - 1 = 0$ for $0 \leq x \leq 180$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	4	2.3	4						2.3.1	1.2.1	Source **1989 P1 qu.7**

•[1] $\sin 3x° = 0.5$
•[2] $3x = 30, 150$
•[3] $x = 10, 50$
•[4] solution is $10, 50, 130$

[SQA] 7.

(a) Show that $2\cos 2x^\circ - \cos^2 x^\circ = 1 - 3\sin^2 x^\circ$. **2**

(b) Hence solve the equation $2\cos 2x^\circ - \cos^2 x^\circ = 2\sin x^\circ$ in the interval $0 \leq x < 360$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(a)	2	2.3			1	1			2.3.3		Source
(b)	4	2.3			1	3			2.3.5		1997 P1 qu.18

- •1 substitute $1 - 2\sin^2 x^\circ$ for $\cos 2x^\circ$
- •2 substitute $1 - \sin^2 x^\circ$ for $\cos^2 x^\circ$
- •3 $3\sin^2 x^\circ + 2\sin x^\circ - 1 = 0$
- •4 $(3\sin x^\circ - 1)(\sin x^\circ + 1) = 0$
- •5 $\sin x^\circ = \frac{1}{3}, -1$
- •6 $19 \cdot 5,\ 160 \cdot 5,\ 270$

[SQA] 8. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	5	2.3	5						2.3.5		Source: 1996 P1 qu.10

- •1 $2\sin x\cos x + \sin x = 0$
- •2 $\sin x(2\cos x + 1) = 0$
- •3 $\sin x = 0, \quad \cos x = -\frac{1}{2}$
- •4 1st: $x = 0,\ 180$
- •5 2nd: $x = 120,\ 240$

[SQA] 9. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3\cos(2x^\circ - 40^\circ) - 1 = 0$. **5**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	5	2.3			5				2.3.1		Source: 1992 P1 qu.5

- •1 $\cos(2x - 40)^\circ = \frac{1}{3}$
- •2 $\cos^{-1}\frac{1}{3} = 70.53$
- •3 $2x - 40 = 70.5 \quad 289.5 \quad 430.5 \quad 649.5$
- •4 $x = 55.25 \quad 164.75 \quad 235.25 \quad 344.75$
- •5 $x = 235.25$

[SQA] 10.

(*a*) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(*b*) Show that there are no real roots for θ. **1**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(a)	3	2.3					1	2	2.3.3	2.1.6	Source
(b)	1	1.2						1	1.2.1		1998 P1 qu.18

•1 $2\cos^2\theta - 1 + 8\cos\theta + 9$

•2 $2(\cos\theta + 2)^2 = 0$

or "$b^2 - 4ac$" $= 16 - 4\times1\times4$

•3 $\cos\theta = -2$ twice or "$b^2 - 4ac$" $= 0$

•4 $\cos\theta = -2$ has no solution

[SQA] 11. If $f(a) = 6\sin^2 a - \cos a$, express $f(a)$ in the form $p\cos^2 a + q\cos a + r$.

Hence solve, correct to three decimal places, the equation $6\sin^2 a - \cos a = 5$ for $0 \leq a \leq \pi$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	4	2.3			2	2			2.3.1		Source
											1993 P1 qu.17

•1 subst. leading from $\sin^2$ *to* $\cos^2$

•2 $-6\cos^2 a - \cos a + 6 = 5$

•3 solving the quadratic

•4 1.231 and 2.094

[SQA] 12. Find the values of t, where $0 < t < 2\pi$, for which $4\cos\left(2t - \frac{\pi}{4}\right)$ has its maximum value. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	4	2.3	4						2.3.1		Source
											1989 P1 qu.15

•1 $\cos\left(2t - \frac{\pi}{4}\right) = 1$

•2 $2t - \frac{\pi}{4} = 0$

•3 $t = \frac{\pi}{8}$

•4 $\frac{\pi}{8}, \frac{9\pi}{8}$

[SQA] 13. Solve the equation $2\sin\left(2x - \frac{\pi}{6}\right) = 1,\ 0 \leq x < 2\pi$. **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	4	2.3	4						2.3.1	1.2.1	Source 1998 P1 qu.9

- $\bullet^1$ $\sin\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}$
- $\bullet^2$ $2x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$ (*accept* 30, 150)
- $\bullet^3$ $x = \frac{\pi}{6}, \frac{\pi}{2}$
- $\bullet^4$ $x = \frac{7\pi}{6}, \frac{3\pi}{2}$

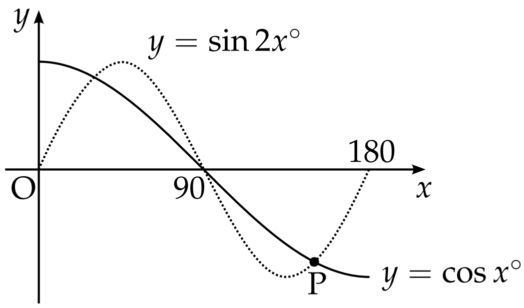
Alternative for 2nd and 3rd marks

- $\bullet^2$ $2x - \frac{\pi}{6} = \frac{\pi}{6},\ x = \frac{\pi}{6}$
- $\bullet^3$ $2x - \frac{\pi}{6} = \frac{5\pi}{6},\ x = \frac{\pi}{2}$

[SQA] 14. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	4	C	NC	T10	$30, 90, 150$	2001 P1 Q5
(*b*)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

- $\bullet^1$ ss: use double angle formula
- $\bullet^2$ pd: factorise
- $\bullet^3$ pd: process
- $\bullet^4$ pd: process
- $\bullet^5$ ic: interpret graph

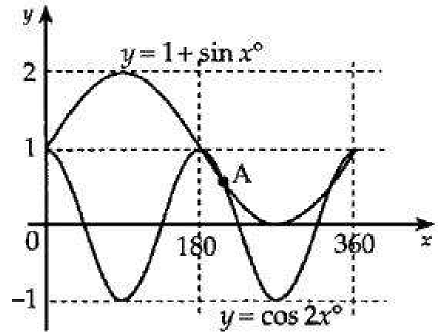
- $\bullet^1$ $2\sin x^\circ \cos x^\circ$
- $\bullet^2$ $\cos x^\circ(2\sin x^\circ - 1)$
- $\bullet^3$ $\cos x^\circ = 0, \sin x^\circ = \frac{1}{2}$
- $\bullet^4$ $90, 30, 150$

or

- $\bullet^3$ $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$
- $\bullet^4$ $\cos x^\circ = 0$ and $x = 90$
- $\bullet^5$ $\left(150, -\frac{\sqrt{3}}{2}\right)$

[SQA] 15. The diagram shows two curves with equations $y = \cos 2x°$ and $y = 1 + \sin x°$ where $0 \leq x \leq 360$.

Find the x-coordinate of the point of intersection at A. **4**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	4	2.3	1	3					2.3.5		Source 1991 P1 qu.20

- •¹ $\cos 2x° = 1 + \sin x°$
- •² $2\sin^2 x° + \sin x° = 0$
- •³ $\sin x° = 0$ *or* $-\frac{1}{2}$
- •⁴ $x = 210$

[SQA] 16. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x°)$, (ii) $2\sin(x°)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0°, 60°, 180°, 300°, 360°$	

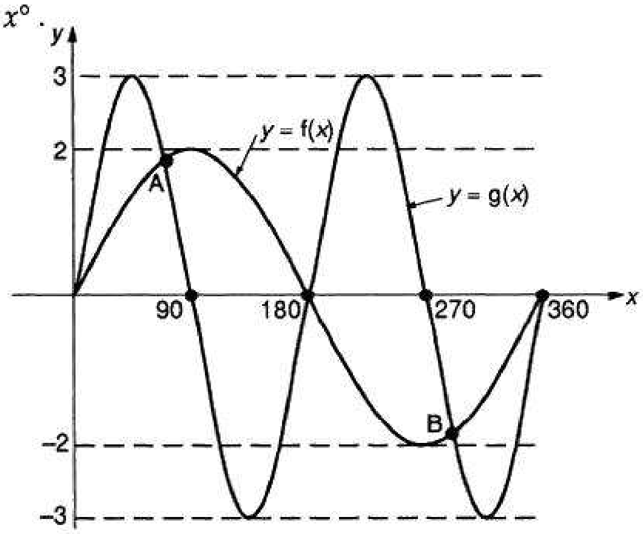
•¹ ic: interpret $f(g(x))$	•¹ $\sin(2x°)$
•² ic: interpret $g(f(x))$	•² $2\sin(x°)$
•³ ss: equate for intersection	•³ $2\sin(2x°) = 2\sin(x°)$
•⁴ ss: substitute for $\sin 2x$	•⁴ appearance of $2\sin(x°)\cos(x°)$
•⁵ pd: extract a common factor	•⁵ $2\sin(x°)\,(2\cos(x°) - 1)$
•⁶ pd: solve a 'common factor' equation	•⁶ $\sin(x°) = 0$ **and** $0, 180, 360$
•⁷ pd: solve a 'linear' equation	•⁷ $\cos(x°) = \frac{1}{2}$ **and** $60, 300$
	or
	•⁶ $\sin(x°) = 0$ **and** $\cos(x°) = \frac{1}{2}$
	•⁷ $0, 60, 180, 300, 360$

[SQA] 17. (*a*) Solve the equation $3\sin 2x° = 2\sin x°$ for $0 \leq x \leq 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions *f* and *g*. State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of *x* in the interval $0 \leq x \leq 360$ for which $3\sin 2x° < 2\sin x°$. **(3)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main	Additional	2.3
(*a*)	4	2.3			4				2.3.5		Source 1992 Paper 2 Qu.7
(*b*)	1	1.2			1				1.2.7		
(*c*)	2	1.2			2				1.2.9		
(*d*)	3	1.2			2	1			1.2.10		

(*a*)
- •1 strategy: ie $\sin 2x = 2\sin x\cos x$
- •2 $\sin x = 0$ AND $\cos x = \frac{1}{3}$
- •3 0, 180 AND 360
- •4 70.5 AND 289.5 AND no other angles

(*b*)
- •5 $f(x) = 2\sin x°$, $g(x) = 3\sin 2x°$

(*c*)
- •6 $x = 70.5$ AND 289.5
- •7 $y = 1.89$ AND -1.89

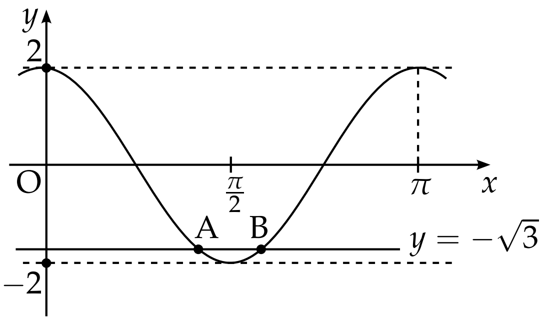
(*d*)
- •8 70.5 AND 180
- •9 289.5 AND 360
- •10 use inequality signs logically to connect the points of intersection (ie **not** for $180 < x < 70.5$)

[SQA] 18. The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

Find the coordinates of B. **3**

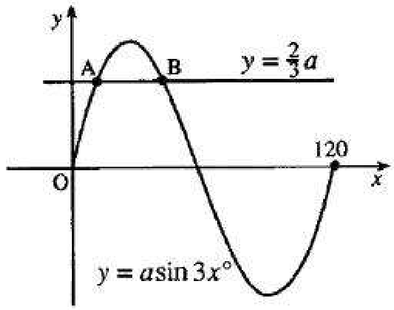


Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	1	C	NC	T4	$y = 2\cos 2x$	2002 P1 Q8
(*b*)	3	C	NC	T7	$B(\frac{7\pi}{12}, -\sqrt{3})$	

•1 ic: interpret graph	•1 $2\cos 2x$
•2 ss: equate equal parts	•1 $2\cos 2x = -\sqrt{3}$
•3 pd: solve linear trig equation in radians	•2 $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$
•4 ic: interpret result	•3 $x = \frac{7\pi}{12}$

[SQA] 19. The diagram shows part of the graph of $y = a\sin 3x°$ and the line with equation $y = \frac{2}{3}a$.

Find the x-coordinates of A and B. **4**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	2.3 Source
.	4	2.3	4						2.3.1	1999 P1 qu.14

•1 $a\sin 3x = \frac{2}{3}a$ *stated or implied by* •2

•2 $\sin 3x = \frac{2}{3}$

•3 $3x = 41.8,\ 138.2$ (138.2 *stated or implied by* 46.1 in •4)

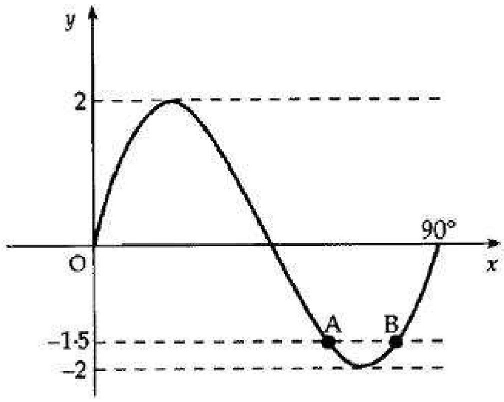
•4 13.9, 46.1

[SQA] 20. The diagram shows the graph of a sine function from 0° to 90°.

(*a*) State the equation of the graph. 2

(*b*) The line with equation $y = -1{\cdot}5$ intersects the curve at A and B.

Find the coordinates of A and B. 3



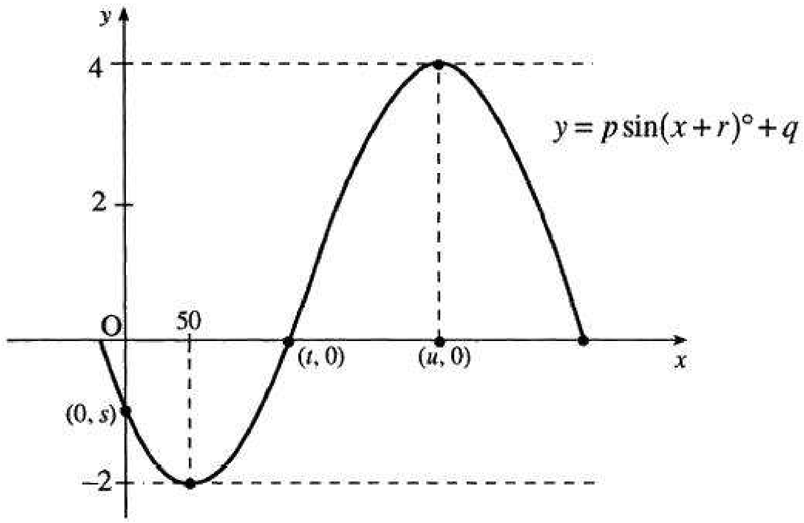
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
											Source
(a)	2	1.2			2				1.2.2	1.2.7	
(b)	3	2.3			3				2.3.1		1990 P1 qu.10

- •¹ $\sin 4x$
- •² (trig function) $\times 2$
- •³ $f(x) = -1 \cdot 5$
- •⁴ $57 \cdot 1°$
- •⁵ $77 \cdot 9°$

[SQA] 21. The sketch represents part of the graph of a trigonometric function of the form $y = p\sin(x+r)° + q$. It crosses the axes at $(0, s)$ and $(t, 0)$, and has turning points at $(50, -2)$ and $(u, 4)$.

(i) Write down values for p, q, r and u. **(4)**

(ii) Find the values for s and t. **(4)**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(a)	4	1.2			2	2			1.2.3		Source
(b)	4	2.3				4			2.3.1		1997 Paper 2 Qu.9

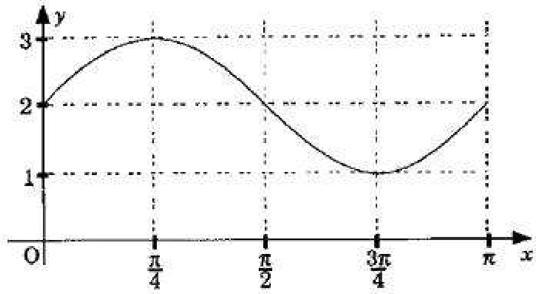
(a)
- •[1] $p = -3$
- •[2] $q = 1$
- •[3] $r = 40$ or -320
- •[4] $u = 230$

(b)
- •[5] replace x by 0
- •[6] -0.928
- •[7] replace y by 0
- •[8] 120.5

[SQA] 22. The diagram shows the graph of the function $y = a + b\sin cx$ for $0 \le x \le \pi$.

(*a*) Write down the values of a, b and c. 3

(*b*) Find algebraically the values of x for which $y = 2{\cdot}5$. 3

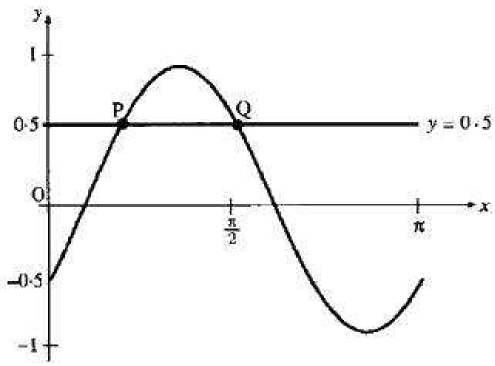


part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
											Source
(a)	3	1.2	3						1.2.3		
(b)	3	2.3	3						2.3.1	1.2.11	1994 P1 qu.12

•¹	$a = 2$	•⁴	$2 + \sin 2x = 2\frac{1}{2}$	OR •⁴	$2 + \sin 2x = 2\frac{1}{2}$
•²	$b = 1$	•⁵	$2x = \frac{\pi}{6},\ \frac{5\pi}{6}$	•⁵	$2x = \frac{\pi}{6},\ x = \frac{\pi}{12}$
•³	$c = 2$	•⁶	$x = \frac{\pi}{12},\ \frac{5\pi}{12}$ $(0{\cdot}262,\ 1{\cdot}309)$	•⁶	$2x = \frac{5\pi}{6}, x = \frac{5\pi}{12}$

[SQA] 23. The diagram shows a sketch of the graph of $y = \sin\left(2x - \frac{\pi}{6}\right)$, $0 \le x \le \pi$, and the straight line $y = 0{\cdot}5$. These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q. 4



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
											Source
.	4	2.3	4						2.3.1	1.2.1	1996 P1 qu.12

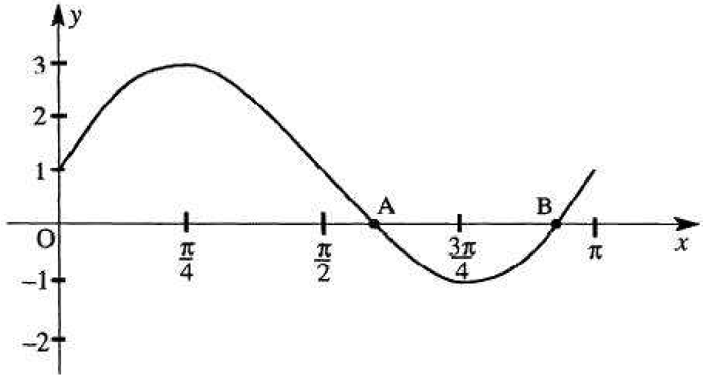
•¹ $\sin\left(2x - \frac{\pi}{6}\right) = 0 \cdot 5$ *stated or implied by 2nd mark*

•² $2x - \frac{\pi}{6} = \frac{\pi}{6}$

•³ $2x - \frac{\pi}{6} = \frac{5\pi}{6}$

•⁴ $\left(\frac{\pi}{6}, 0\cdot 5\right), \left(\frac{\pi}{2}, 0\cdot 5\right)$

[SQA] 24. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \le x \le \pi$.



(*a*) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. **(5)**

(*b*) The points $(0, 2)$ and $(\pi, 0)$ are joined by a straight line l. In how many points does l intersect the given graph ? **(1)**

(*c*) C is the point on the given graph with an x-coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l. **(3)**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(*a*)	5	2.3	3	2					2.3.1		Source
(*b*)	1	0.1	1						0.1		1993 Paper 2
(*c*)	3	0.1		3					0.1		Qu.6

(*a*) •[1] $2\sin 2x + 1 = 0$

•[2] $\sin 2x = -\frac{1}{2}$

•[3] for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$

•[4] $\left(\frac{7\pi}{12}, 0\right)$

•[5] $\left(\frac{11\pi}{12}, 0\right)$

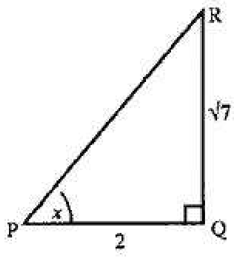
(*b*) •[6] 3

(*c*) •[7] $y_C = 1$

•[8] for a strategy to make a decision about C

•[9] for making a consistent decision about C

[SQA] 25. Using triangle PQR, as shown, find the exact value of $\cos 2x$. **3**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	3	2.3	3						2.3.3		Source 1999 P1 qu.12

•[1] $\cos x = \frac{2}{\sqrt{11}}$ *or* $\sin x = \frac{\sqrt{7}}{\sqrt{11}}$

•[2] $\cos 2x = 2 \times \left(\frac{2}{\sqrt{11}}\right)^2 - 1$

•[3] $-\frac{3}{11}$

[SQA] 26. If $\cos\theta = \frac{4}{5}$, $0 \leq \theta < \frac{\pi}{2}$, find the exact value of

(*a*) $\sin 2\theta$ **2**

(*b*) $\sin 4\theta$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
(a)	2	2.3	2						2.3.3		Source
(b)	3	2.3		3					2.3.3		1994 P1 qu.13

•[1] $\sin\theta = \frac{3}{5}$

•[2] $\frac{24}{25}$

•[3] $2\sin 2\theta \cos 2\theta$

•[4] $\cos 2\theta = \frac{7}{25}$

•[5] $\frac{336}{625}$

[SQA] 27. Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Content Reference: Additional	2.3
.	3	2.3	3						2.3.3		Source 1995 P1 qu.12

•[1] "third side" $= \sqrt{20}$

•[2] $\sin\alpha = \frac{\sqrt{11}}{\sqrt{20}}$ *or* $\cos\alpha = \frac{3}{\sqrt{20}}$

•[3] $2 \times \frac{\sqrt{11}}{\sqrt{20}} \times \frac{3}{\sqrt{20}}$

[SQA] 28. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	3	2.3	3						2.3.3		Source: **1990 P1 qu.9**

- •[1] *strat for exact value: e.g.* $\sin^2 D = 1 - \cos^2 D$
- •[2] $\sin D = \frac{1}{\sqrt{5}}$
- •[3] $\cos 2D = \frac{3}{5}$

[SQA] 29. Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the exact value of $\sin 2A$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	3	2.3	3						2.3.3		Source: **1991 P1 qu.12**

- •[1] *strat for* cos: *eg* $\cos^2 = 1 - \sin^2$
- •[2] $\cos A = \frac{\sqrt{7}}{4}$
- •[3] $\sin 2A = \frac{3\sqrt{7}}{8}$

[SQA] 30. For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$.

Show that the exact value of $\sin(P+Q)$ is $\frac{63}{65}$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	3	2.3	3						2.3.2		Source: **1993 P1 qu.6**

- •[1] $\cos P = \frac{5}{13}$
- •[2] $\cos Q = \frac{4}{5}$
- •[3] $\frac{12}{13} \times \frac{4}{5} + \frac{5}{13} \times \frac{3}{5}$

[SQA] 31. Find the exact value of $\sin\theta° + \sin(\theta° + 120°) + \cos(\theta° + 150°)$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3 Source
.	3	2.3	3						2.3.2		1994 P1 qu.6

- •[1] $\sin\theta\cos 120 + \cos\theta\sin 120$ **and** $\cos\theta\cos 150 - \sin\theta\sin 150$
- •[2] correct use of exact values
- •[3] simplification to zero

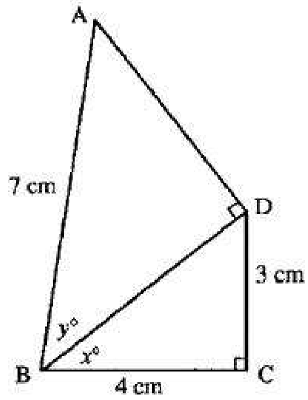
[SQA] 32. If $x°$ is an acute angle such that $\tan x° = \frac{4}{3}$, show that the exact value of $\sin(x° + 30°)$ is $\dfrac{4\sqrt{3}+3}{10}$. **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3 Source
.	3	2.3	3						2.3.2		1997 P1 qu.7

- •[1] $\sin x° \cos 30° + \cos x° \sin 30°$
- •[2] $\sin x° = \frac{4}{5}$ & $\cos x° = \frac{3}{5}$
- •[3] $\frac{4}{5}\cdot\frac{\sqrt{3}}{2} + \frac{3}{5}\cdot\frac{1}{2}$ and completes proof

[SQA] 33. The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$.

Show that the exact value of $\cos(x+y)°$ is $\frac{20-6\sqrt{6}}{35}$. **3**



part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
.	3	2.3					3		2.3.2		Source 1996 P1 qu.15

- •¹ know to calculate missing sides
- •² BD = 5, AD = $\sqrt{24}$
- •³ $\cos x \cos y - \sin x \sin y = \frac{4}{5}.\frac{5}{7} - \frac{3}{5}.\frac{\sqrt{24}}{7}$

[SQA] 34. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

Find the exact value of

(*a*) $\sin 2A$ **2**

(*b*) $\cos 2A$ **1**

(*c*) $\sin(2A+B)$. **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(a)	2	2.3	2						2.3.3		Source
(b)	1	2.3	1						2.3.3		1998 P1 qu.7
(c)	2	2.3	2						2.3.2		

- •¹ $\sin A = \frac{3}{5}$ and $\cos A = \frac{4}{5}$
- •² $\sin 2A = 2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$ (*accept* 0.96)
- •³ $\cos 2A =$ *e.g.* $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$ (*accept* 0.28)
- •⁴ $\sin 2A \cos B + \cos 2A \sin B$
- •⁵ $\sin B = \frac{5}{13}$ *and* $\cos B = \frac{12}{13}$ *and* $\frac{323}{325}$

[SQA] 35. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$.

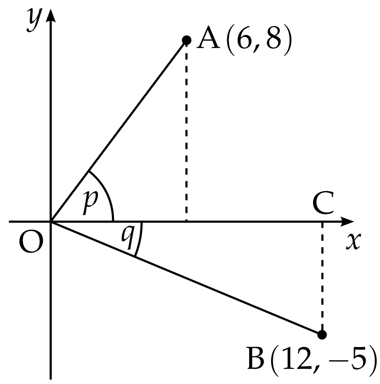
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \le x \le 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x + \frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•1 ic: interpret composite functions	•1 $\sin(x + \frac{\pi}{4})$
•2 ic: interpret composite functions	•2 $\cos(x + \frac{\pi}{4})$
•3 ss: expand $\sin(x + \frac{\pi}{4})$	•3 $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete**
•4 ic: interpret	•4 $g(h(x)) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$
•5 ic: substitute	•5 $(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x) - (\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x)$
•6 pd: start solving process	•6 $\frac{2}{\sqrt{2}}\sin x$
•7 pd: process	•7 $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 36. On the coordinate diagram shown, A is the point $(6,8)$ and B is the point $(12,-5)$. Angle AOC $= p$ and angle COB $= q$.

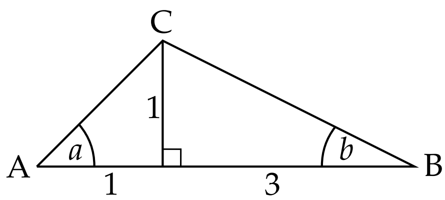
Find the exact value of $\sin(p+q)$. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	$\frac{63}{65}$	2000 P1 Q1

•[1] ss: know to use trig expansion •[2] pd: process missing sides •[3] ic: interpret data •[4] pd: process	•[1] $\sin p \cos q + \cos p \sin q$ •[2] 10 **and** 13 •[3] $\frac{8}{10}\cdot\frac{12}{13}+\frac{6}{10}\cdot\frac{5}{13}$ •[4] $\frac{126}{130}$

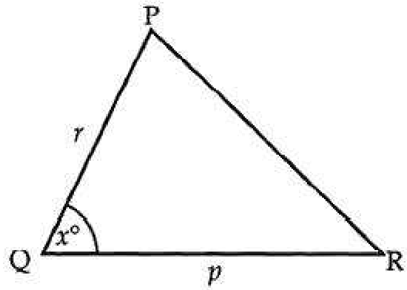
[SQA] 37. In triangle ABC, show that the exact value of $\sin(a+b)$ is $\dfrac{2}{\sqrt{5}}$. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	proof	2002 P1 Q5

•[1] pd: process the missing sides •[2] ss: expand •[3] pd: substitute •[4] pc: process and complete proof	•[1] $AC=\sqrt{2}$ **and** $BC=\sqrt{10}$ *stated or implied by* •[3] •[2] $\sin(a+b)=\sin a\cos b+\cos a\sin b$ •[3] $\frac{1}{\sqrt{2}}\cdot\frac{3}{\sqrt{10}}+\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{10}}$ •[4] $\frac{4}{\sqrt{20}}=\ldots=\frac{2}{\sqrt{5}}$

[SQA] 38. The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = $x°$.



(*a*) Show that $\frac{\sin x°}{p} = \frac{\sin 2x°}{r}$. **(3)**

(*b*) (i) State the value of $x°$ when $p = r$.

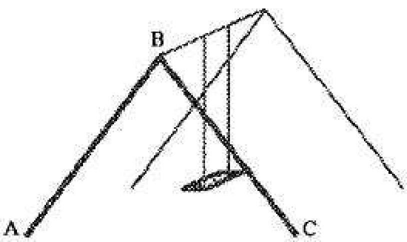
(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of $x°$. **(5)**

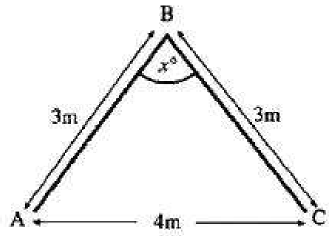
part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main	Additional	2.3
(*a*)	3	0.1					2	1	0.1		Source
(*b*)	5	2.3					5		2.3.5,	0.1	1991 Paper 2 Qu. 3

(*a*)
- •[1] $(180 - 2x)°$
- •[2] $\frac{\sin x°}{p} = \frac{\sin(180 - 2x)°}{r}$
- •[3] $\sin(180 - 2x)° = \sin 2x°$ **stated explicitly**

(*b*)
- •[4] $60°$
- •[5] $\sin x° = \sin 2x°$
- •[6] $\sin x°(2\cos x° - 1) = 0$
- •[7] $\sin x° = 0$ *and* $\cos x° = \frac{1}{2}$
- •[8] $x = 60$ is only answer **stated explicitly**

[SQA] 39. The framework of a child's swing has dimensions as shown in the diagram on the right.
Find the exact value of $\sin x°$. **5**





part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	2.3 Source
.	5	2.3	1	4					2.3.4	1996 P1 qu.18

•1	sketch with $\frac{x}{2}$ marked in r / a Δ	**OR**	•1	know to use cosine rule
•2	height of triangle $= \sqrt{5}$		•2	$\cos x = \frac{3^2+3^2-4^2}{2.3.3}$
•3	$\sin x = 2\sin\frac{1}{2}x\cos\frac{1}{2}x$		•3	$\frac{1}{9}$
•4	$\sin\frac{x}{2} = \frac{2}{3}$ *and* $\cos\frac{1}{2}x = \frac{\sqrt{5}}{3}$		•4	draw r / a Δ *or* use $\cos^2 x + \sin^2 x = 1$
•5	$\sin x = \frac{4\sqrt{5}}{9}$		•5	$\sin x = \frac{\sqrt{80}}{9}$

[END OF WRITTEN QUESTIONS]