# Vectors Past Papers Unit 3 outcome 1

## Written Questions

[SQA] 1. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find

(a) the components of vector $\overrightarrow{AB}$; 1

(b) the length of AB. 2

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 1 | 3.1 | | | | | 1 | | 3.1.1 | | Source |
| (b) | 2 | 3.1 | | | | | 2 | | 3.1.3 | | 1993 P1 qu.1 |

•1 $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

•2 $\sqrt{(-3+1)^2 + (2-3)^2 + (4-2)^2}$

•3 3

[SQA] 2. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined by

$$\boldsymbol{p} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}, \quad \boldsymbol{q} = \boldsymbol{i} + 4\boldsymbol{k} \quad \text{and} \quad \boldsymbol{r} = 4\boldsymbol{i} - 3\boldsymbol{j}.$$

(a) Express $\boldsymbol{p} - \boldsymbol{q} + 2\boldsymbol{r}$ in component form. 2

(b) Calculate $\boldsymbol{p}.\boldsymbol{r}$ 1

(c) Find $|\boldsymbol{r}|$. 1

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 2 | 3.1 | | | | | 2 | | 3.1.1 | | Source |
| (b) | 1 | 3.1 | | | | | 1 | | 3.1.9 | | 1998 P1 qu.3 |
| (c) | 1 | 3.1 | | | | | 1 | | 3.1.1 | | |

•1 $p = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, q = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, r = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ s/i by •2

•2 $\begin{pmatrix} 8 \\ -5 \\ -5 \end{pmatrix}$

•3 1

•4 5

[SQA] 3. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows:

$$\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},\ \boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},\ \boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}.$$

(*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. **1**

(*b*) Find the value of $|2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}|$. **2**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Source |
| (a) | 1 | 3.1 | | | | | 1 | | 3.1.8 | | 1989 P1 qu.3 |
| (b) | 2 | 3.1 | | | | | 2 | | 3.1.1 | | |

•[1] $6i - 7j + 6k$

•[2] $\sqrt{6^2 + (-7)^2 + 6^2}$

•[3] 11

[SQA] 4. The vector $ai + bj + k$ is perpendicular to both the vectors $i - j + k$ and $-2i + j + k$. Find the values of $a$ and $b$. 3

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Source |
| . | 3 | 3.1 | | | | | 3 | | 3.1.8 | | 1990 P1 qu.12 |

•[1] $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = a - b + 1$ *or* $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} . \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = -2a + b + 1$

•[2] $a - b + 1 = 0$ *or* $-2a + b + 1 = 0$

•[3] $a = 2$ *and* $b = 3$

[SQA] 5. The position vectors of the points P and Q are $\boldsymbol{p} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 7\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. 2

(*b*) Find the length of PQ. 1

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | Source |
| (a) | 2 | 3.1 | | | | | 2 | | 3.1.8 | 3.1.1 | 1997 P1 qu.4 |
| (b) | 1 | 3.1 | | | | | 1 | | 3.1.3 | | |

•[1] $q - p = 8i - 4j + k$

*or* $p = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix},\ q = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$

•[2] $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$

•[3] 9

[SQA] 6. Calculate the length of the vector $2\mathbf{i}-3\mathbf{j}+\sqrt{3}\mathbf{k}$. 2

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 2 | 3.1 | | | | | 2 | | 3.1.8 | | Source 1995 P1 qu.1 |

•1 $\sqrt{2^2+(-3)^2+(\sqrt{3})^2}$ stated or implied by •2

•2 4

[SQA] 7. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are defined as follows:

$$\boldsymbol{a}=2\boldsymbol{i}-\boldsymbol{k},\quad \boldsymbol{b}=\boldsymbol{i}+2\boldsymbol{j}+\boldsymbol{k},\quad \boldsymbol{c}=-\boldsymbol{j}+\boldsymbol{k}.$$

(*a*) Evaluate $\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$. 3

(*b*) From your answer to part (*a*), make a deduction about the vector $\boldsymbol{b}+\boldsymbol{c}$. 2

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 3 | 3.1 | | | | | 3 | | 3.1.8 | 3.1.9 | Source |
| (b) | 2 | 3.1 | | | | | | 2 | 3.1.10 | | 1993 P1 qu.12 |

•1 $\boldsymbol{a}=\begin{pmatrix}2\\0\\-1\end{pmatrix}, \boldsymbol{b}=\begin{pmatrix}1\\2\\1\end{pmatrix}, \boldsymbol{c}=\begin{pmatrix}0\\-1\\1\end{pmatrix}$

•2 $\boldsymbol{a}.\boldsymbol{b}=1$

•3 $\boldsymbol{a}.\boldsymbol{c}=-1$

•4 $\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}=\boldsymbol{a}.(\mathbf{b}+\mathbf{c})$

•5 $\boldsymbol{a}\perp\mathbf{b}+\mathbf{c}$

[SQA] 8. Show that the vectors $\boldsymbol{a}=2\boldsymbol{i}+3\boldsymbol{j}-\boldsymbol{k}$ and $\boldsymbol{b}=3\boldsymbol{i}-\boldsymbol{j}+3\boldsymbol{k}$ are perpendicular. 3

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.8 | 3.1.1 | Source 1991 P1 qu.3 |

•1 *strat:* $\boldsymbol{a}.\boldsymbol{b}=\ldots\ldots\ldots$

•2 $\boldsymbol{a}.\boldsymbol{b}=0\Rightarrow$ perpendicularity explicitly stated

•3 $\begin{pmatrix}2\\3\\-1\end{pmatrix}.\begin{pmatrix}3\\-1\\3\end{pmatrix}=6-3-3=0$

[SQA] 9. If $\boldsymbol{u} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$, write down the components of $\boldsymbol{u} - \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$. Hence show that $\boldsymbol{u} + \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are perpendicular. 3

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.2 | 3.1.1 | Source 1994 P1 qu.7 |

•[1] $u + v = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$ and $u - v = \begin{pmatrix} -4 \\ -2 \\ 4 \end{pmatrix}$

•[2] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v}) = 8 - 16 + 8$

•[3] $(\mathbf{u}+\mathbf{v}).(\mathbf{u}-\mathbf{v}) = 0$ so $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are perpendicular

[SQA] 10. (a) Show that the points L(–5, 6, –5), M(7, –2, –1) and N(10, –4, 0) are collinear. 4

(b) Find the ration in which M divides LN. 1

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 4 | 3.1 | | | | | 4 | | 3.1.7 | | Source |
| (b) | 1 | 3.1 | | | | | 1 | | 3.1.6 | | 1991 P1 qu.7 |

•[1] $\vec{LM} = \begin{pmatrix} 12 \\ -8 \\ 4 \end{pmatrix}$ or equivalent combinations for (a)

•[2] $\vec{MN} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

•[3] $\vec{LM} = 4\vec{MN}$

•[4] vectors are parallel and have common point so L, M, N are collinear

•[5] 4:1

[SQA] 11. Show that P(2, 2, 3), Q(4, 4, 1) and R(5, 5, 0) are collinear and find the ratio in which Q divides PR. 4

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 3.1 | | | | | 4 | | 3.1.7 | 3.1.6 | Source 1990 P1 qu.4 |

•[1] $\vec{PQ} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$

•[2] $\vec{QR} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \frac{1}{2}\vec{PQ}$

or equivalent

•[3] vectors parallel and have pt in common so pts collinear

•[4] $PQ:QR = 2:1$

[SQA] 12. A is the point (2, −5, 6), B is (6, −3, 4) and C is (12, 0, 1). Show that A, B and C are collinear and determine the ratio in which B divides AC. **4**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Content Reference: Additional | 3.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 3.1 | | | | | 4 | | 3.1.7 | 3.1.6 | 1996 P1 qu.6 |

| | |
|---|---|
| •1 $\overrightarrow{AB}=\begin{pmatrix}4\\2\\-2\end{pmatrix}$ *or* $\overrightarrow{AC}=\begin{pmatrix}10\\5\\-5\end{pmatrix}$ *or* $\overrightarrow{BC}=\begin{pmatrix}6\\3\\-3\end{pmatrix}$ | •3 $AB \parallel BC$ **and** B is point in common |
| •2 $\overrightarrow{AB}=2\begin{pmatrix}2\\1\\-1\end{pmatrix}$ *and* $\overrightarrow{BC}=3\begin{pmatrix}2\\1\\-1\end{pmatrix}$ or equivalent | •4 2:3 (*or* equivalent e.g. $1:1\frac{1}{2}$) |

[SQA] 13. The point Q divides the line joining $P(-1,-1,0)$ to $R(5,2,-3)$ in the ratio $2:1$.

Find the coordinates of Q. **3**

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| | 3 | C | NC | G25 | $(3,1,-2)$ | 2002 P1 Q2 |

| | |
|---|---|
| •1 pd: find vector components<br>•2 ss: use parallel vectors<br>•3 pd: process vectors | •1 $\overrightarrow{PR}=\begin{pmatrix}6\\3\\-3\end{pmatrix}$<br>•2 $\overrightarrow{PQ}=\frac{2}{3}\overrightarrow{PR}$<br>•3 $Q=(3,1,-2)$ |

[SQA] 14. For what value of $t$ are the vectors $\boldsymbol{u}=\begin{pmatrix}t\\-2\\3\end{pmatrix}$ and $\boldsymbol{v}=\begin{pmatrix}2\\10\\t\end{pmatrix}$ perpendicular? **2**

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| | 2 | C | CN | G27 | $t=4$ | 2000 P2 Q7 |

| | |
|---|---|
| •1 ss: know to use scalar product<br>•2 ic: interpret scalar product | •1 $\boldsymbol{u}.\boldsymbol{v}=2t-20+3t$<br>•2 $\boldsymbol{u}.\boldsymbol{v}=0 \Rightarrow t=4$ |

[SQA] 15. $A(4,4,10)$, $B(-2,-4,12)$ and $C(-8,0,10)$ are the vertices of a right-angled triangle.

Determine which angle of the triangle is the right angle. **3**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.10 | Source 1989 P1 qu.6 |

•1 $\vec{AB}=\begin{pmatrix}-6\\-8\\2\end{pmatrix}, \vec{BC}=\begin{pmatrix}-6\\4\\-2\end{pmatrix}, \vec{AC}=\begin{pmatrix}-12\\-4\\0\end{pmatrix}$

•2 $|\vec{AC}|$ is longest so $\vec{AB}.\vec{CB}=-36+32+4=0$

•3 $A\hat{B}C=90°$

[SQA] 16. Find the value of $k$ for which the vectors $\begin{pmatrix}1\\2\\-1\end{pmatrix}$ and $\begin{pmatrix}-4\\3\\k-1\end{pmatrix}$ are perpendicular. 3

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.10 | Source 1995 P1 qu.4 |

•1 $\begin{pmatrix}1\\2\\-1\end{pmatrix}.\begin{pmatrix}-4\\3\\k-1\end{pmatrix}=0$

•2 $1\times-4+2\times3+-1(k-1)$

•3 3

[SQA] 17. PQRS is a parallelogram with vertices $P(1,3,3)$, $Q(4,-2,-2)$ and $R(3,1,1)$.

Find the coordinates of S. **3**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.4 | Source 1989 P1 qu.4 |

•1 $\vec{QP}=\begin{pmatrix}-3\\5\\5\end{pmatrix}$

•2 $R=(3,1,1)$ and $\vec{RS}=\begin{pmatrix}-3\\5\\5\end{pmatrix}$ stated or implied by •3

•3 $S=(0,6,6)$

[SQA] 18. ABCD is a quadrilateral with vertices $A(4,-1,3)$, $B(8,3,-1)$, $C(0,4,4)$ and $D(-4,0,8)$.

(*a*) Find the coordinates of M, the midpoint of AB. **1**

(*b*) Find the coordinates of the point T, which divides CM in the ratio 2 : 1. **3**

(*c*) Show that B, T and D are collinear and find the ratio in which T divides BD. **4**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 1 | 0.1 | | | | | 1 | | 0.1 | Source |
| (*b*) | 3 | 3.1 | | | | | 3 | | 3.1.6 | 1989 Paper 2 |
| (c) | 4 | 3.1 | | | | | 4 | | 3.1.7, 3.1.6 | Qu. 2 |

(*a*) •[1] (6,1,1)

(*b*) •[2] e.g. $\overrightarrow{CM}=\begin{pmatrix}6\\-3\\-3\end{pmatrix}$

•[3] $\overrightarrow{CT}=\begin{pmatrix}4\\-2\\-2\end{pmatrix}$

•[4] $T=(4,2,2)$

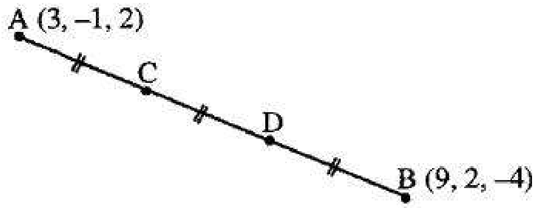
(*c*) •[5] e.g. $\overrightarrow{BT}=\begin{pmatrix}-4\\-1\\3\end{pmatrix}$

•[6] $\overrightarrow{TD}=\begin{pmatrix}-8\\-2\\6\end{pmatrix}=2\times\overrightarrow{BT}$

•[7] TD is parallel to BT, T is common point so B, T, D collinear

•[8] BT:TD = 1:2

[SQA] 19. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).




(*a*) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(*b*) Find the coordinates of C and D. 2

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 2 | 3.1 | | | | | 2 | | 3.1.1 | Source |
| (b) | 2 | 3.1 | | | | | 2 | | 3.1.1 | 1998 P1 qu.5 |

•[1] $\overrightarrow{AB}=\begin{pmatrix}6\\3\\-6\end{pmatrix}$

•[2] $\overrightarrow{AC}=\begin{pmatrix}2\\1\\-2\end{pmatrix}$

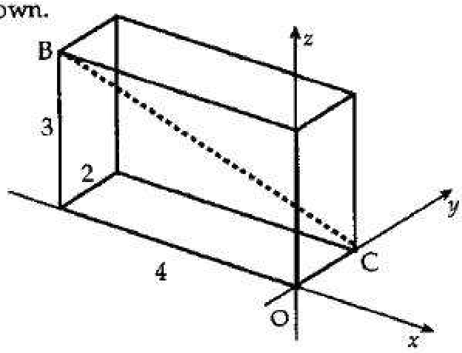
•[3] $C=(5,0,0)$

•[4] $D=(7,1,-2)$

[SQA] 20. A cuboid crystal is placed relative to the coordinate axes as shown.

(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $\left|\overrightarrow{BC}\right|$. 2

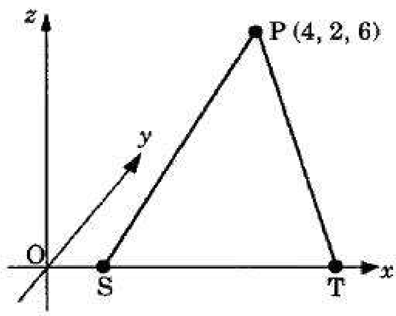



| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 1 | 3.1 | | | | | 1 | | 3.1.1 | | Source |
| (b) | 1 | 3.1 | | | | | 1 | | 3.1.3 | | 1990 P1 qu.5 |

•[1] $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

•[2] $\sqrt{29}$

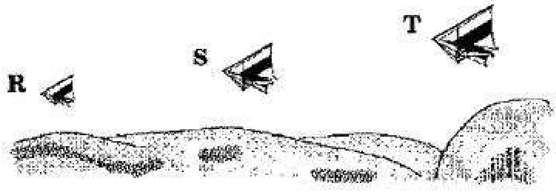
[SQA] 21. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 3




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | | 3 | 3.1.3 | | Source |
| | | | | | | | | | | | 1994 P1 qu.18 |

| | | | | | |
|---|---|---|---|---|---|
| •[1] | $(x, 0, 0)$ or equiv. | OR •[1] | $PQ = \sqrt{40}$ | OR •[1] | $d^2 = 7^2 - 6^2 - 2^2$ |
| •[2] | $(x-4)^2 + 4 + 36 = 49$ or equiv. | •[2] | $d = 3$ | •[2] | $d = 3$ |
| •[3] | $x = 1,\ 7$ | •[3] | $(1,0,0),\ (7,0,0)$ | •[3] | $(1,0,0),\ (7,0,0)$ |

[SQA] 22. Relative to the top of a hill, three gliders have positions given by R(–1, –8, –2), S(2, –5, 4) and T(3, –4, 6).
Prove that R, S and T are collinear. 3



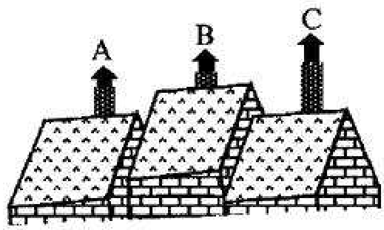

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | | Source 1994 P1 qu.4 |

•[1] $\overrightarrow{ST} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ or equivalent **and** $\overrightarrow{RS} = \begin{pmatrix}3\\3\\6\end{pmatrix}$ or equivalent

•[2] $\overrightarrow{RS} = 3\overrightarrow{ST}$ or equiv.

•[3] RS // ST **and** S is common.

[SQA] 23. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B(2, –1, 4) and C(4, –9, 8).
Show that A, B and C are collinear. 3



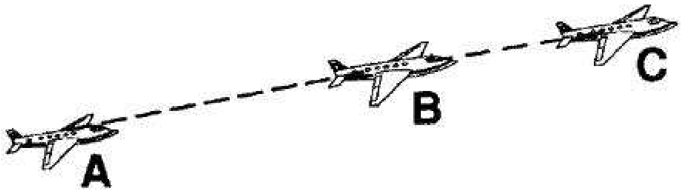

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.7 | Source 1997 P1 qu.2 |

•[1] $\overrightarrow{AB} = \begin{pmatrix}1\\-4\\2\end{pmatrix}$

•[2] $\overrightarrow{BC} = \begin{pmatrix}2\\-8\\4\end{pmatrix}$ AND $\overrightarrow{BC} = 2 \times \overrightarrow{AB}$

•[3] $\overrightarrow{AB} \,||\, \overrightarrow{BC}$ & B is common hence A, B, C collinear

[SQA] 24. An aircraft flying at a constant speed on a straight flight path takes 2 minutes to fly from A to B and 1 minute to fly from B to C. Relative to a suitable set of axes, A is the point $(-1, 3, 4)$ and B is the point $(3, 1, -2)$. Find the co-ordinates of the point C. 3




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.6 | | Source: 1992 P1 qu.15 |

•1 $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$

•2 $\overrightarrow{BC} = \overrightarrow{AB}$

•3 $(5, 0, -5)$

[SQA] 25. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

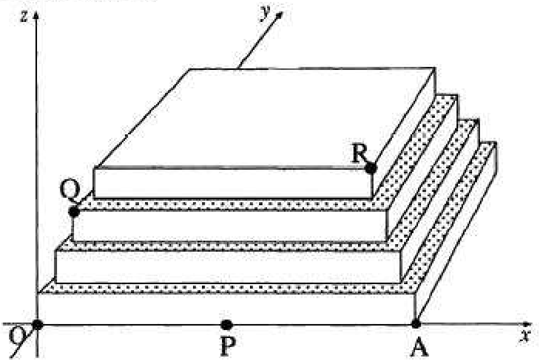



Diagram 1

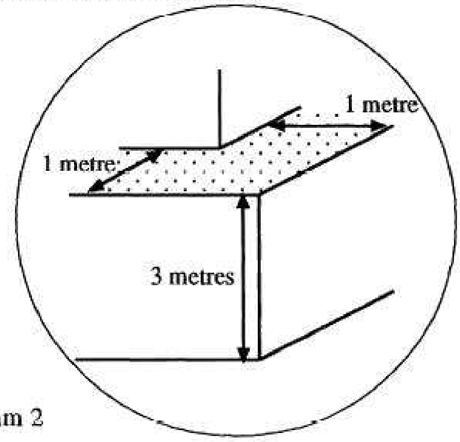



Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**

(*b*) Find the size of angle QPR. **(7)**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 2 | 3.1 | | | 2 | | | | 3.1.1 | | 1996 Paper 2 Qu.3 |
| (*b*) | 7 | 3.1 | | | 7 | | | | 3.1.11 | | |

**(a)** •[1] $Q = (2,2,9)$

•[2] $R = (21,3,12)$

**(b)** •[3] $\cos\theta = \frac{a.b}{|a||b|}$ *with some subsequent use*

*eg* $\cos Q\hat{P}R = \frac{\overrightarrow{PQ}.\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$

•[4] $\overrightarrow{PQ} = \begin{pmatrix} -10 \\ 2 \\ 9 \end{pmatrix}$ •[5] $\overrightarrow{PR} = \begin{pmatrix} 9 \\ 3 \\ 12 \end{pmatrix}$

•[6] $|\overrightarrow{PQ}| = \sqrt{185}$

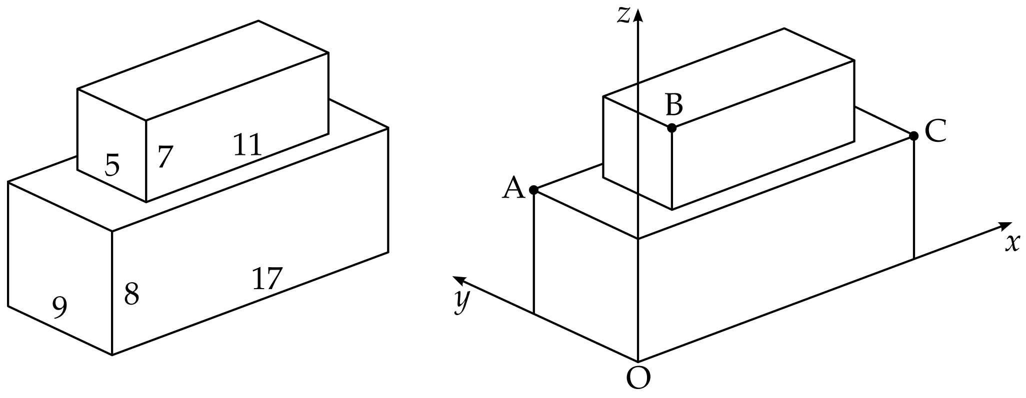
•[7] $|\overrightarrow{PR}| = \sqrt{234}$

•[8] $\overrightarrow{PQ}.\overrightarrow{PR} = 24$

•[9] $Q\hat{P}R = 83\cdot4^\circ$

[SQA] 26. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.




(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| (*a*) | 1 | C | CN | G22 | B$(3, 2, 15)$ | 2000 P2 Q9 |
| (*b*) | 6 | C | CR | G28 | 92·5° | |

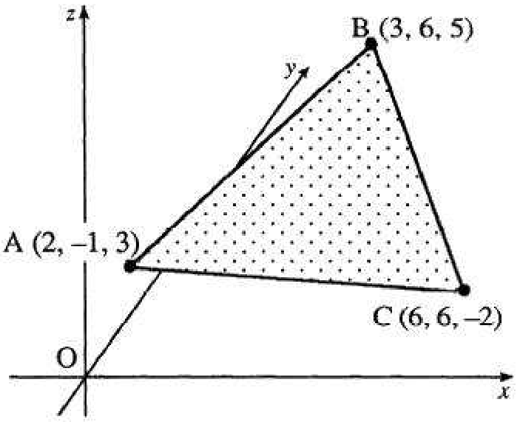
| | |
|---|---|
| •[1] ic: interpret 3-d representation<br>•[2] ss: know to use scalar product<br>•[3] pd: process vectors<br>•[4] pd: process vectors<br>•[5] pd: process lengths<br>•[6] pd: process scalar product<br>•[7] pd: evaluate scalar product | •[1] B$=(3,2,15)$ *treat* $\begin{pmatrix}3\\2\\15\end{pmatrix}$ *as bad form*<br>•[2] $\cos A\hat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$<br>•[3] $\overrightarrow{BA} = \begin{pmatrix}-3\\7\\-7\end{pmatrix}$<br>•[4] $\overrightarrow{BC} = \begin{pmatrix}14\\-2\\-7\end{pmatrix}$<br>•[5] $\lvert\overrightarrow{BA}\rvert = \sqrt{107}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{249}$<br>•[6] $\overrightarrow{BA}.\overrightarrow{BC} = -7$<br>•[7] $A\hat{B}C = 92{\cdot}5°$ |

[SQA] 27. A triangle ABC has vertices
A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).

(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 2 | 3.1 | | | 2 | | | | 3.1.1 | | Source |
| (*b*) | 5 | 3.1 | | | 5 | | | | 3.1.11 | | 1998 Paper 2 |
| (*c*) | 2 | 0.1 | | | 2 | | | | 0.1 | | Qu. 1 |

**(*a*)** •[1] $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•[2] $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

**(*b*)** •[3] $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •4 to •7*

•[4] $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•[5] $\overrightarrow{AB} = \sqrt{54}$

•[6] $\overrightarrow{AC} = \sqrt{90}$

•[7] $B\hat{A}C = 51\cdot 9°$

**(*c*)** •[8] **identify** 2 sides and included angle *e.g.* $\sqrt{54}$, $\sqrt{90}$, $B\hat{A}C$

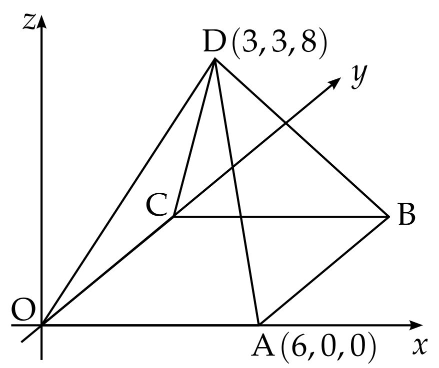
•[9] $27\cdot 4$

[SQA] 28. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are $(6, 0, 0)$ and $(3, 3, 8)$.

C lies on the $y$-axis.




(*a*) Write down the coordinates of B. **1**

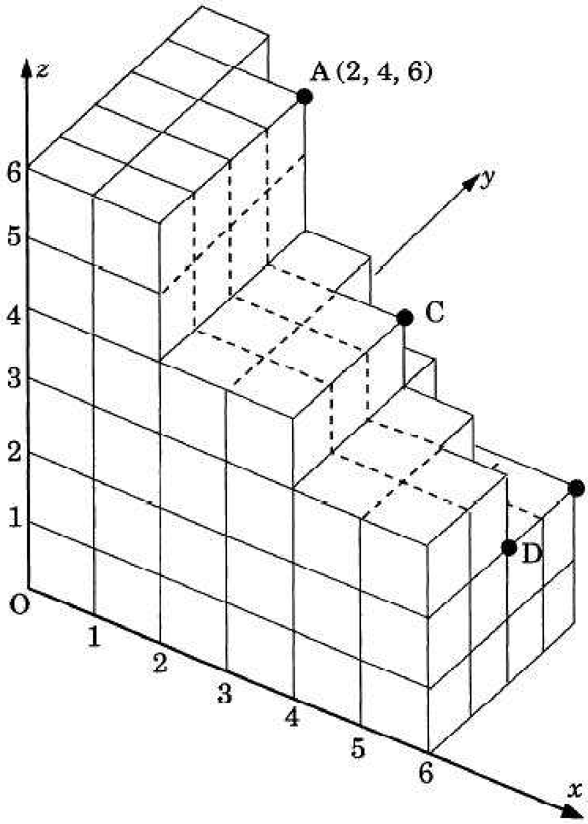
(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| (*a*) | 1 | C | CN | G22 | $(6, 6, 0)$ | 2002 P2 Q2 |
| (*b*) | 2 | C | CN | G17 | $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$, $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$ | |
| (*c*) | 4 | C | CR | G28 | $38{\cdot}7°$ | |

| | |
|---|---|
| •[1] ic: interpret diagram | •[1] $B = (6, 6, 0)$ |
| •[2] ic: write down components of a vector | •[2] $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$ |
| •[3] ic: write down components of a vector | •[3] $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$ |
| •[4] ss: use e.g. scalar product formula | •[4] $\cos A\hat{D}B = \frac{\overrightarrow{DA}.\overrightarrow{DB}}{\lvert\overrightarrow{DA}\rvert\lvert\overrightarrow{DB}\rvert}$ |
| •[5] pd: process lengths | •[5] $\lvert\overrightarrow{DA}\rvert = \sqrt{82}$, $\lvert\overrightarrow{DB}\rvert = \sqrt{82}$ |
| •[6] pd: process scalar product | •[6] $\overrightarrow{DA}.\overrightarrow{DB} = 64$ |
| •[7] pd: process angle | •[7] $A\hat{D}B = 38{\cdot}7°$ |

[SQA] 29. With coordinate axes as shown, the point A is (2,4,6).




(*a*) Write down the coordinates of B,C and D. **(3)**

(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(*d*) Hence calculate the size of angle OAB. **(2)**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 3 | 3.1 | | | 3 | | | | 3.1.1 | Source |
| (*b*) | 1 | 3.1 | | | 1 | | | | 3.1.6 | 1994 Paper 2 |
| (*c*) | 4 | 3.1 | | | 4 | | | | 3.1.11 | Qu.3 |
| (*d*) | 2 | 0.1 | | | 2 | | | | 0.1 | |

(*a*) •1 One of B,C or D

•2 Remaining two of B, C and D

•3 B(6,4,2), C(4,3,4), D(6,2,2)

(*b*) •4 $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(*c*) •5 $\cos A\hat{O}B = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$ *or* $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•6 $\overrightarrow{OA}.\overrightarrow{OB} = 40$ *or* $AB^2 = 32$

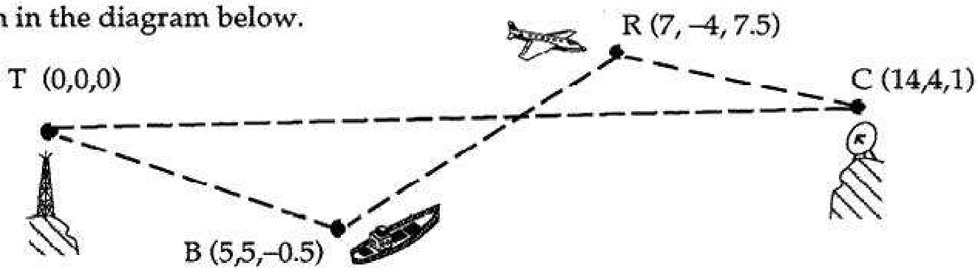
•7 $OA = \sqrt{56} = OB$

•8 $44^\circ$

(*d*) •9 strategy: e.g. use isosceles Δ

•10 $68^\circ$

[SQA] 30. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.




The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 3 | 3.1 | | | 3 | | | | 3.1.3 | | Source |
| (*b*) | 2 | 3.1 | | | 2 | | | | 3.1.3 | | **1992 Paper 2** |
| (*c*) | 3 | 3.1 | | | 3 | | | | 3.1.10 | | **Qu.2** |
| (d) | 5 | 3.1 | | | 5 | | | | 3.1.11 | | |

(*a*) •1 Strategy: use vectors or 3-D distance formula

•2 $\overrightarrow{BR}=\begin{pmatrix}2\\7\\4\end{pmatrix}$ *or* $BR^2=2^2+7^2+4^2$

•3 answer

(*b*) •4 $|\overrightarrow{MR}|=\sqrt{115.25}$ or equivalent

•5 answer

(*c*) •6 know to use a scalar product

•7 $\overrightarrow{TC}.\overrightarrow{BR}=0$

•8 communication: 0 ⇔ perpendicularity

(*d*) •9 Strategy: know to use

$\cos T\hat{C}R=\dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

•10 $\overrightarrow{TC}=\begin{pmatrix}12\\-4\\1\end{pmatrix}$ and $\overrightarrow{RC}=\begin{pmatrix}5\\-6\\-2\end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\overrightarrow{TC}.\overrightarrow{RC}=82$

•13 36.7°

[SQA] 31. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

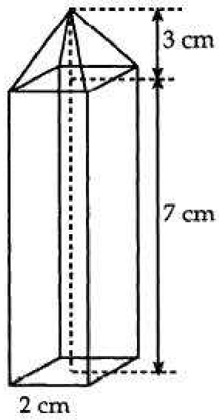


Diagram 1

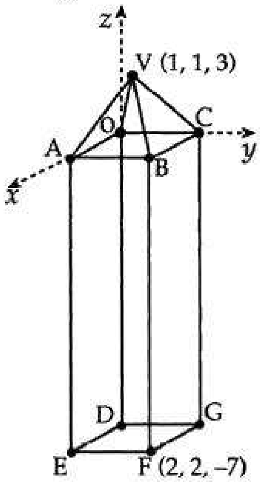


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 7 | 3.1 | | | | | 7 | | 3.1.11, | 3.1.1 | Source **1991 Paper 2 Qu. 5** |
| (*b*) | 3 | 0.1 | | | | | 3 | | 0.1 | | |

(*a*) •[1] $\overrightarrow{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•[2] $E = (2, 0, -7)$

•[3] $\overrightarrow{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•[4] $\cos E\hat{V}F = \frac{\overrightarrow{VE}.\overrightarrow{VF}}{|\overrightarrow{VE}|\,|\overrightarrow{VF}|}$ This may appear as $\frac{100}{102}$ after the completion of •[5] and •[6].

•[5] $\overrightarrow{VE}.\overrightarrow{VF} = 100$

•[6] $|\overrightarrow{VE}|\,|\overrightarrow{VF}| = 102$

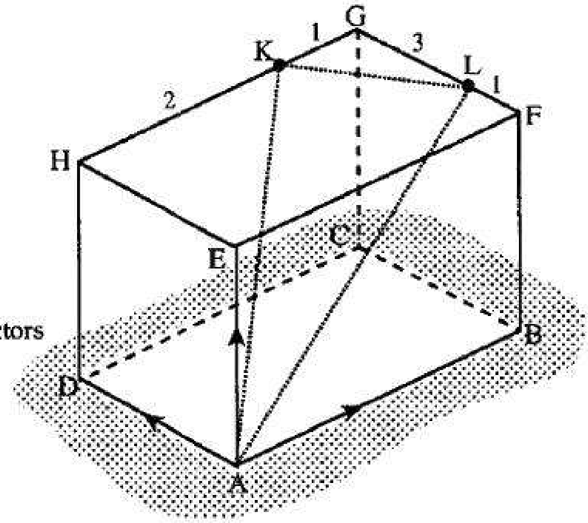
•[7] $11.4°$

(*b*) •[8] $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•[9] $\frac{1}{2} \times 102 \times \sin 11.4°$

•[10] $10.02$

[SQA] 32. ABCDEFGH is a cuboid.




K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ can be represented by the vectors

$\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\begin{pmatrix}-8\\4\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-3\\5\end{pmatrix}$ respectively.

(a) Calculate the components of $\overrightarrow{AK}$. **2**

(b) Calculate the components of $\overrightarrow{AL}$. **2**

(c) Calculate the size of angle KAL. **5**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 2 | 3.1 | | | | | 2 | | 3.1.2 | | Source |
| (b) | 2 | 3.1 | | | | | 2 | | 3.1.2 | | 1999 Paper 2 |
| (c) | 5 | 3.1 | | | | | 5 | | 3.1.11 | | Qu. 3 |

(a) •¹ obtaining for example $\begin{pmatrix}2\\4\\2\end{pmatrix}$

•² $\overrightarrow{AK} = \begin{pmatrix}-5\\5\\11\end{pmatrix}$

(b) •³ obtaining for example $\begin{pmatrix}-2\\1\\1\end{pmatrix}$

•⁴ $\overrightarrow{AL} = \begin{pmatrix}2\\4\\9\end{pmatrix}$

(c) •⁵ *strategy e.g.* $\cos K\hat{A}L = \frac{\overrightarrow{AK}.\overrightarrow{AL}}{|AK|\times|AL|}$

•⁶ 109

•⁷ $\sqrt{171}$

•⁸ $\sqrt{101}$

•⁹ $\hat{A} = 34.0$

*OR*

•⁵ *strategy e.g.* $\cos K\hat{A}L = \frac{AK^2+AL^2-KL^2}{2AK\times AL}$

•⁶ $\sqrt{54}$

•⁷ $\sqrt{171}$

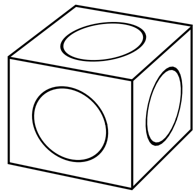
•⁸ $\sqrt{101}$

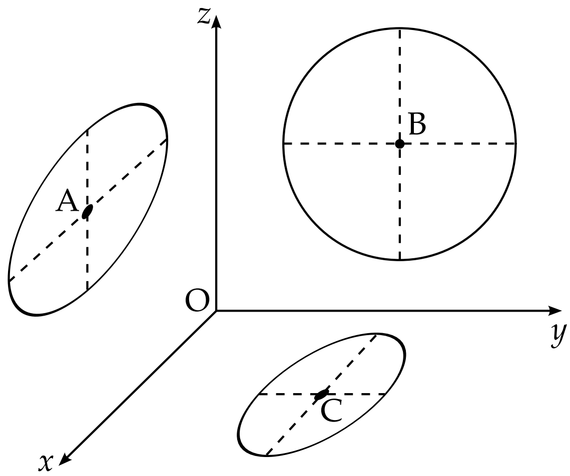
•⁹ $\hat{A} = 34.0$

[SQA] 33. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6,0,7)$, $B(0,5,6)$ and $C(4,5,0)$.

Find the size of angle ABC. **7**





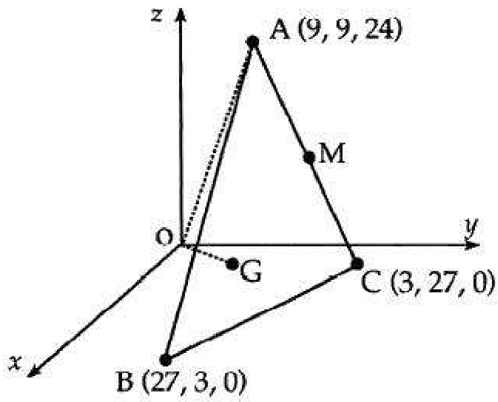

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| | 5 | C | CR | G17, G16, G22 | | 2001 P2 Q4 |
| | 2 | A/B | CR | G26, G28 | $71{\cdot}5^\circ$ | |

| | |
|---|---|
| •[1] ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$<br>•[2] ic: state vector e.g. $\overrightarrow{BA}$<br>•[3] ic: state a consistent vector e.g. $\overrightarrow{BC}$<br>•[4] pd: process $\lvert\overrightarrow{BA}\rvert$<br>•[5] pd: process $\lvert\overrightarrow{BC}\rvert$<br>•[6] pd: process scalar product<br>•[7] pd: find angle | •[1] use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ *stated or implied by •[7]*<br>•[2] $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$<br>•[3] $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$<br>•[4] $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$<br>•[5] $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$<br>•[6] $\overrightarrow{BA}.\overrightarrow{BC} = 18$<br>•[7] $A\hat{B}C = 71{\cdot}5^\circ$ |

[SQA] 34. (*a*) Relative to mutually perpendicular axes O*x*, O*y* and O*z*, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.

Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**

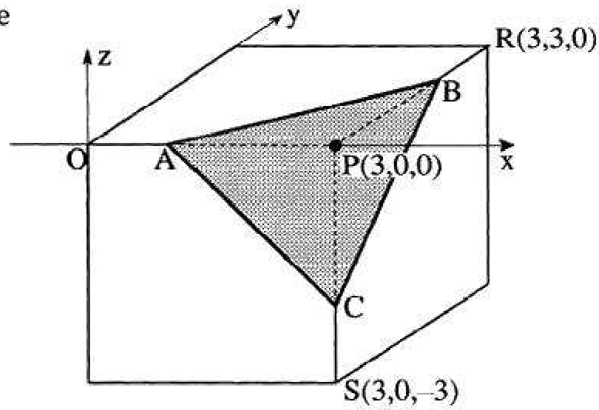



| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 3 | 3.1 | | | 3 | | | | 3.1.6 | Source 1990 Paper 2 Qu. 4 |
| (*b*) | 5 | 3.1 | | | 5 | | | | 3.1.11 | |

(*a*)
- •[1] M = (6, 18, 12)
- •[2] *e.g.* $\overrightarrow{BG} = \frac{2}{3}\begin{pmatrix}-21\\15\\12\end{pmatrix}$
- •[3] G = (13, 13, 8)

(*b*)
- •[4] $\cos A\hat{O}G = \dfrac{\overrightarrow{OA}.\overrightarrow{OG}}{|\overrightarrow{OA}||\overrightarrow{OG}|}$
- •[5] $\overrightarrow{OA} = \begin{pmatrix}9\\9\\24\end{pmatrix}$ and $\overrightarrow{OG} = \begin{pmatrix}13\\13\\8\end{pmatrix}$
- •[6] $\overrightarrow{OA}.\overrightarrow{OG} = 426$
- •[7] $|\overrightarrow{OA}| = \sqrt{738}$ and $|\overrightarrow{OG}| = \sqrt{402}$
- •[8] 38.5°

[SQA] 35. A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.
The point A divides OP in the ratio 1:2.
Points B and C similarly divide RP and SP respectively in the ratio 1:2.




(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

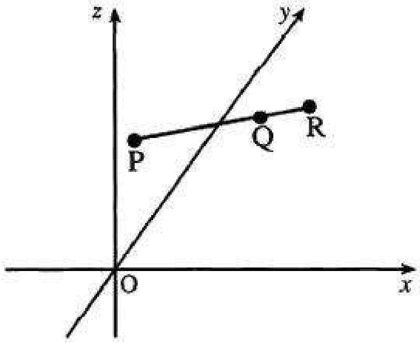
| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference : Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 3 | 3.1 | | | 3 | | | | 3.1.6 | | Source |
| (*b*) | 4 | 3.1 | | | 4 | | | | 3.1.3, | 0.1 | 1993 Paper 2 |
| (*c*) | 5 | 0.1 | | | 5 | | | | 0.1 | | Qu.5 |

**(a)**
- •[1] $A(1,0,0)$
- •[2] $B(3,2,0)$
- •[3] $C(3,0,-2)$

**(b)**
- •[4] strategy for area of triangle and attempt to calculate parts
- •[5] $60°$ or altitude $=\sqrt{6}$
- •[6] side $=2\sqrt{2}$
- •[7] using chosen formula correctly

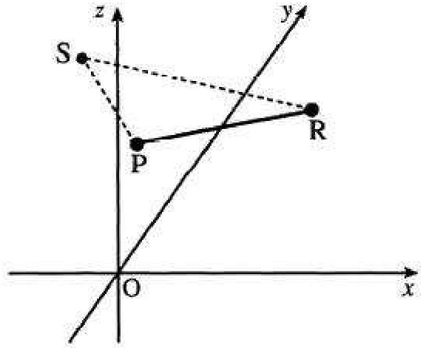
**(c)**
- •[8] 54 unit$^2$ for cube
- •[9] know how to calculate s.a of crystal
- •[10] area of 1 pentagonal face = 7 unit$^2$
- •[11] 51.5 unit$^2$ for crystal $(48+2\sqrt{3})$
- •[12] strategy for finding % decrease

[SQA] 36. Relative to the axes shown and with an appropriate scale, P(−1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.




(*a*) Find the coordinates of R. **(3)**

(*b*) Roads from P and R are built to meet at the point S (−2, 2, 5). Calculate the size of angle PSR. **(7)**




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 3 | 3.1 | | | 3 | | | | 3.1.6 | | Source |
| (*b*) | 7 | 3.1 | | | 7 | | | | 3.1.11 | | 1997 Paper 2 Qu.2 |

**(*a*)**

•1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$   •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•3 $R = (7, -1, 6)$

**(*b*)**

•4 $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

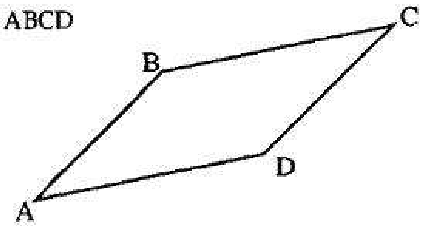
•5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$   •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•7 $|SP| = \sqrt{11}$   •8 $|SR| = \sqrt{91}$

•9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•10 $P\hat{S}R = 84 \cdot 6°$

[SQA] 37. A is the point $(2, -1, 4)$, B is $(7, 1, 3)$ and C is $(-6, 4, 2)$. If ABCD is a parallelogram, find the coordinates of D. 3




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Content Reference: Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.4 | 3.1.1 | Source 1994 P1 qu.3 |

| | |
|---|---|
| •1 $\vec{OD} = \vec{OA} + \vec{AD}$ or equivalent, stated or implied by •3 | •1 $\vec{OD} = \vec{OM} + \vec{MD}$, M is midpoint of AC |
| •2 $\vec{BC} = \begin{pmatrix} -13 \\ 3 \\ -1 \end{pmatrix}$ *or* $\vec{CB}$ or $\vec{AB}$ or $\vec{BA}$ | *OR* •2 $\vec{BM} = \begin{pmatrix} -9 \\ \frac{1}{2} \\ 0 \end{pmatrix}$ |
| •3 D = (−11, 2, 3) | •3 $D = (-11, 2, 3)$ |

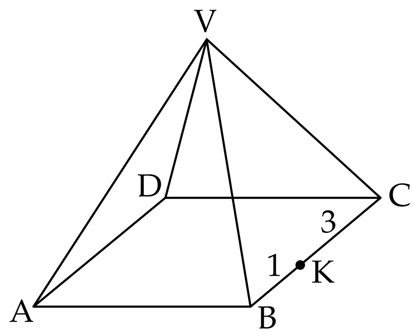
[SQA] 38. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.




K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form. **3**

| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| | 3 | C | CN | G25, G21, G20 | $\begin{pmatrix}1\\-8\\-16\end{pmatrix}$ | 2000 P1 Q7 |

| | |
|---|---|
| •1 ss: recognise crucial aspect<br>•2 ic: interpret ratio<br>•3 pd: process components | •1 $\overrightarrow{VK} = \overrightarrow{VA} + \overrightarrow{AB} + \overrightarrow{BK}$ **or** $\overrightarrow{VK} = \overrightarrow{VB} + \overrightarrow{BK}$<br>•2 $\overrightarrow{BK} = \frac{1}{4}\overrightarrow{BC}$ **or** $\frac{1}{4}\overrightarrow{AD}$ **or** $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **or** $\begin{pmatrix}-1\\-7\\-17\end{pmatrix}$<br>•3 $\overrightarrow{VK} = \begin{pmatrix}1\\-8\\-16\end{pmatrix}$ |

[SQA] 39. VABCD is a pyramid with rectangular base ABCD.

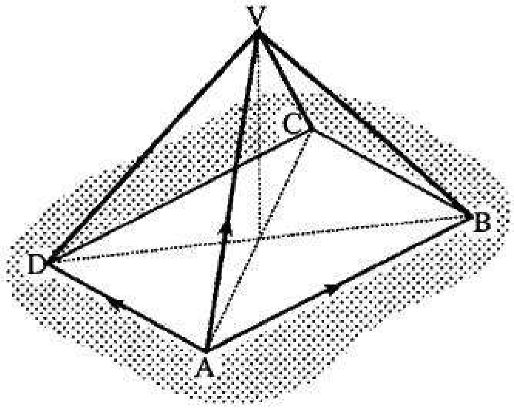
The vectors $\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$\overrightarrow{AB} = 8i + 2j + 2k$

$\overrightarrow{AD} = -2i + 10j - 2k$ and

$\overrightarrow{AV} = i + 7j + 7k.$

Express $\overrightarrow{CV}$ in component form.




3

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main Additional | 3.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.8 | 1999 P1 qu.6 |

•[1] pathway for $\overrightarrow{CV}$ : $\overrightarrow{CV} = \overrightarrow{CA} + \overrightarrow{AV}$

•[2] *e.g.* $\overrightarrow{CB} = 2i - 10j + 2k$

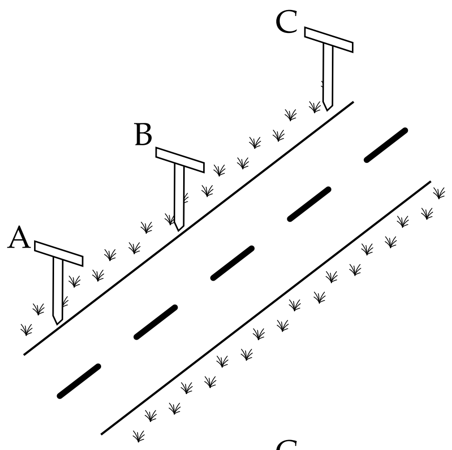
*or* $\overrightarrow{BA} = -8i - 2j - 2k$

*or* $\overrightarrow{AC} = 6i + 12j$

•[3] $\begin{pmatrix} -5 \\ -5 \\ 7 \end{pmatrix}$

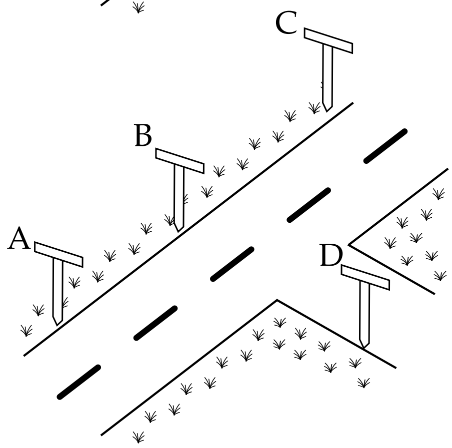
[SQA] 40. (*a*) Roadmakers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points $A(-8,-10,-2)$, $B(-2,-1,1)$ and $C(6,11,5)$.

Determine whether or not the section of road ABC has been built in a straight line. **3**




(*b*) A further T-rod is placed such that D has coordinates $(1,-4,4)$.

Show that DB is perpendicular to AB. **3**




| Part | Marks | Level | Calc. | Content | Answer | U3 OC1 |
|---|---|---|---|---|---|---|
| (*a*) | 3 | C | CN | G23 | the road ABC is straight | 2001 P1 Q3 |
| (*b*) | 3 | C | CN | G27, G17 | proof | |

•[1] ic: interpret vector (e.g. $\overrightarrow{AB}$)
•[2] ic: interpret multiple of vector
•[3] ic: complete proof

•[4] ic: interpret vector (i.e. $\overrightarrow{BD}$)
•[5] ss: state requirement for perpend.
•[6] ic: complete proof

•[1] e.g. $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$

•[2] e.g. $\overrightarrow{BC} = \begin{pmatrix} 8 \\ 12 \\ 4 \end{pmatrix} = \frac{4}{3}\overrightarrow{AB}$ **or**

$\overrightarrow{AB} = 3\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{BC} = 4\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

•[3] a common direction exists **and** a common point exists, so A, B, C collinear

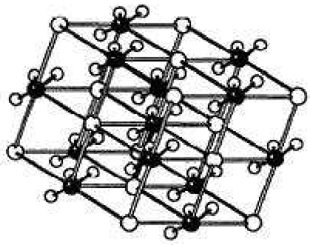
•[4] $\overrightarrow{BD} = \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$

•[5] $\overrightarrow{AB}.\overrightarrow{BD} = 0$
•[6] $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9 = 0$

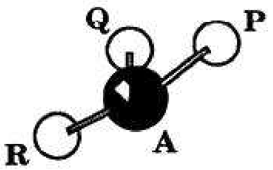
**or**

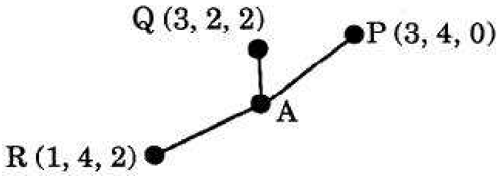
•[5] $\overrightarrow{AB}.\overrightarrow{BD} = 18 - 27 + 9$
•[6] $\overrightarrow{AB}.\overrightarrow{BD} = 0$ so AB is at right angles to BD

[SQA] 41. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.




(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (*a*) | 4 | 3.1 | | | | | 4 | | 3.1.11 | | Source |
| (*b*) | 6 | 3.1 | | | | | 6 | | 3.1.6, | 3.1.3 | 1995 Paper 2 |
| (*c*) | 2 | 3.1 | | | | | 1 | 1 | 3.1.3, | 0.1 | Qu.5 |

**(a)**

•1 $PQ = \sqrt{8}$, $RQ = \sqrt{8}$,

•2 Use s.p.: $\overrightarrow{PQ}.\overrightarrow{RQ} = |\overrightarrow{PQ}|.|\overrightarrow{RQ}|\cos\theta$

•3 $\begin{pmatrix}0\\-2\\2\end{pmatrix}.\begin{pmatrix}2\\-2\\0\end{pmatrix} = 4$

•4 $60^\circ$

**(b)**

•5 $M = (2,3,2)$

•6 $\overrightarrow{PT} = \frac{2}{3}\overrightarrow{PM}$ or equivalent

•7 $\overrightarrow{PT} = \frac{2}{3}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ or equiv.

•8 $T = \left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$

•9 $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ stated or implied

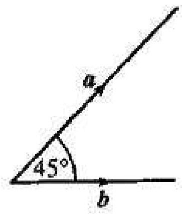
•10 $PT = 2\sqrt{\frac{2}{3}}, QT = 2\sqrt{\frac{2}{3}}, RT = 2\sqrt{\frac{2}{3}}$ or equivalent

**(c)**

•11 $PA = QA = RA = \sqrt{3}$

•12 A is in a different plane

[SQA] 42. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, with $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2\sqrt{2}$.
These vectors are inclined at an angle of 45° to each other.



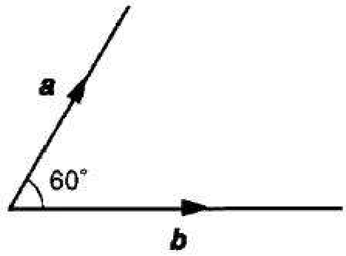

(*a*) Evaluate (i) $\boldsymbol{a.a}$
(ii) $\boldsymbol{b.b}$
(iii) $\boldsymbol{a.b}$ 2

(*b*) Another vector $\boldsymbol{p}$ is defined by $\boldsymbol{p} = 2\boldsymbol{a} + 3\boldsymbol{b}$.
Evaluate $\boldsymbol{p.p}$ and hence write down $|\boldsymbol{p}|$. 4

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (a) | 2 | 3.1 | | | | | 2 | | 3.1.9 | | Source |
| (b) | 4 | 3.1 | | | | | | 4 | 3.1.9 | | 1999 P1 qu.17 |

•[1] $a.a = 9$ and $b.b = 8$
•[2] $a.b = 6$
•[3] $(2a + 3b).(2a + 3b)$
•[4] $4a.a + 9b.b + 12a.b$
•[5] 180
•[6] $\sqrt{180}$

[SQA] 43. The diagram shows representatives of two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, inclined at an angle of 60°.
If $|\boldsymbol{a}| = 2$ and $|\boldsymbol{b}| = 3$, evaluate $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b})$ 3



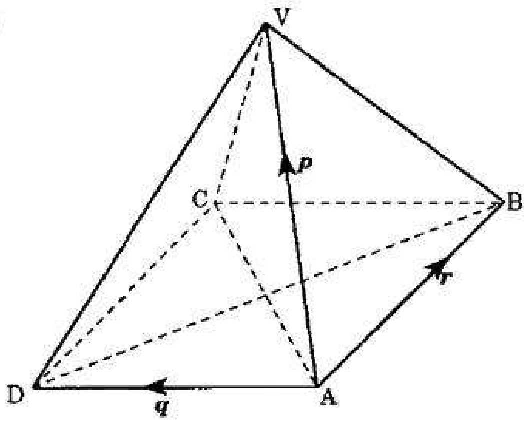

| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 3 | 3.1 | | | | | 3 | | 3.1.9 | | Source 1992 P1 qu.18 |

•[1] $\boldsymbol{a.a + a.b}$
•[2] $2 \times 3 \times \cos 60°$
•[3] 4

[SQA] 44. In the square-based pyramid, all the eight edges are of length 3 units.

$\overrightarrow{AV} = \boldsymbol{p}$, $\overrightarrow{AD} = \boldsymbol{q}$, $\overrightarrow{AB} = \boldsymbol{r}$.

Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$. 4




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 3.1 | | | | | 1 | 3 | 3.1.9 | | 1995 P1 qu.16 |

•[1] $\boldsymbol{p}.\boldsymbol{q}+\boldsymbol{p}.\boldsymbol{r}$

•[2] $V\hat{A}D = 60^\circ$ or equiv.

•[3] $|\boldsymbol{p}||\boldsymbol{q}|\cos V\hat{A}D + |\boldsymbol{p}||\boldsymbol{r}|\cos V\hat{A}B$

•[4] 9

•[1] $r = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, q = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$

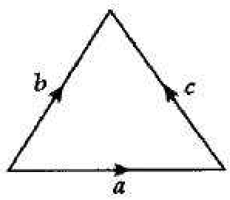
•[2] $p = \begin{pmatrix} -\frac{3}{2} \\ \frac{3}{2} \\ \frac{3}{\sqrt{2}} \end{pmatrix}$

•[3] $\left(-\frac{3}{2}\right)\times(-3)+\left(\frac{3}{2}\right)\times 3+\frac{3}{\sqrt{2}}\times 0$

•[4] 9

[SQA] 45. The sides of this equilateral triangle are 2 units long and represent the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as shown.

Evaluate $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$. 5




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 Source |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 3.1 | 1 | 4 | | | | | 3.1.9 | | 1989 P1 qu.9 |

•[1] $\boldsymbol{a}.\boldsymbol{a}+\boldsymbol{a}.\boldsymbol{b}+\boldsymbol{a}.\boldsymbol{c}$

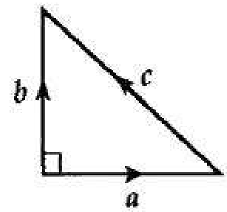
•[2] $a.a = |a||a|\cos 0$

•[3] $a.b = |a||b|\cos 60$

•[4] $a.c = |a||c|\cos 120$

•[5] 4

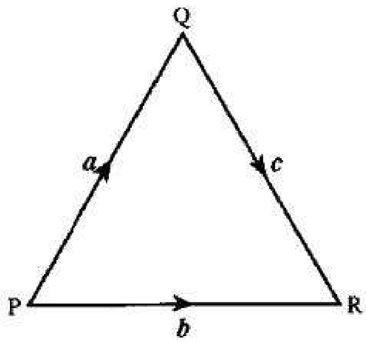
[SQA] 46. The diagram shows a right-angled isosceles triangle whose sides are represented by the vectors **a**, **b** and **c**.
The two equal sides have length 2 units.
Find the value of $\boldsymbol{b}.(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})$ 5




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 5 | 3.1 | | | | | 5 | | 3.1.9 | 3.1.1 | Source 1991 P1 qu.17 |

•[1] $\boldsymbol{b.a}+\boldsymbol{b.b}+\boldsymbol{b.c}$
•[2] $\boldsymbol{b.a}=0$
•[3] $\boldsymbol{b.b}=4$
•[4] $|\boldsymbol{c}|=2\sqrt{2}$
•[5] $\boldsymbol{b.c}=4$

[SQA] 47. PQR is an equilateral triangle of side 2 units.
$\overrightarrow{PQ}=\boldsymbol{a}$, $\overrightarrow{PR}=\boldsymbol{b}$ and $\overrightarrow{QR}=\boldsymbol{c}$.
Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular. 4




| part | marks | Unit | non-calc C | non-calc A/B | calc C | calc A/B | calc neut C | calc neut A/B | Content Reference: Main | Additional | 3.1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| . | 4 | 3.1 | | | | | 1 | 3 | 3.1.9 | 3.1.1 | Source 1997 P1 qu.13 |

•[1] $\boldsymbol{a.b}+\boldsymbol{a.c}$
•[2] $\boldsymbol{a.b}=2\times2\times\frac{1}{2}$
•[3] $\boldsymbol{a.c}=2\times2\times-\frac{1}{2}$
•[4] 0 and $\boldsymbol{a}$ is perpendicular to $(\boldsymbol{b}+\boldsymbol{c})$

*[END OF WRITTEN QUESTIONS]*