^x_y **Quest**

Wave Fctn Past Papers Unit 3 Outcome 4

Written Questions

[SQA]   1. Express $8\cos x° - 6\sin x°$ in the form $k\cos(x° + a°)$ where $k > 0$ and $0 < a < 360$.   **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	4	C	CR	T13	$10\cos(x° + 36·9°)$	2001 P2 Q5

• ^{1} ss: expand $k\cos(x° + a°)$ • ^{2} ic: compare coefficients • ^{3} pd: process • ^{4} pd: process	• ^{1} $k\cos x\cos a - k\sin x\sin a$ *stated explicitly* • ^{2} $k\cos a = 8$ **and** $k\sin a = 6$ *stated explicitly* • ^{3} $k = 10$ • ^{4} $a = 36·9$

[SQA]   2. Express $2\sin x° - 5\cos x°$ in the form $k\sin(x - \alpha)°$, $0 \le \alpha < 360$ and $k > 0$.   **4**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	3.4
.	4	3.4			4				3.4.1	Source 1997 P1 qu.11

• ^{1} $k\sin(x - a) = k\sin x\cos a - k\cos x\sin a$ **stated explicitly**
• ^{2} $k\cos a = 2$ and $k\sin a = 5$
• ^{3} $k = \sqrt{29}$
• ^{4} $a = 68·2$

[SQA]   3. (a) Express $\sin x° - 3\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a.   **4**

(b) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs.   **2**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference: Main Additional	3.4
(a)	4	3.4			4				3.4.1	Source
(b)	2	3.4			1	1			3.4.3	1992 P1 qu.7

• ^{1} $k\cos a = 1$ • ^{2} $k\sin a = 3$ • ^{3} $k = \sqrt{10}$ • ^{4} $a = 71.6$	• ^{5} maximum $= 5 + \sqrt{10}$ • ^{6} angle $= 161.6°$

[SQA] 4 Solve the simultaneous equations $k \sin x° = 5$

$k \cos x° = 2$ where $k \geq 0$ and $0 \leq x \leq 360$ 4

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	4	3.4			4					3.4.1	Source 1995 P1 qu.13

- •¹ $\tan x = \frac{5}{2}$
- •² $x = 68 \cdot 2$
- •³ $k^2 = 25 + 4$ or $k = \frac{5}{\sin 68 \cdot 2}$
- •⁴ $k = \sqrt{29}$

- •¹ $k^2 \left(\sin^2 x + \cos^2 x \right) = 29$
- •² $k = \sqrt{29}$
- •³ $\tan x = \frac{5}{2}$ or $\sin x = \frac{5}{\sqrt{29}}$
- •⁴ $x = 68 \cdot 2$

[SQA] 5 Solve the equation $2 \sin x° - 3 \cos x° = 2 \cdot 5$ in the interval $0 \leq x < 360$. 8

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
.	8	3.4				8				3.4.2	Source 1999 Paper 2 Qu. 9

- •¹ strategy:e.g. ......$k \sin(x - a)$ *stated or implied by •⁶*
- •² $k \sin x \cos a - k \cos x \sin a$ *stated explicitly*
- •³ $k \cos a = 2$ and $k \sin a = 3$ *stated explicitly*
- •⁴ $k = \sqrt{13}$
- •⁵ $a = 56.3$
- •⁶ $\sin(x - 56.3) = \frac{2.5}{\sqrt{13}}$
- •⁷ $x - 56.3 = 43.9, 136.1$ *136.1 stated or implied by the appearance of 192.4 in •⁸*
- •⁸ $100.2°$ and $192.4°$

OR

- •⁷ $x - 56.3 = 43.9, x = 100.2°$
- •⁸ $192.4°$

$k \cos(x - a)$
$k \cos x \cos a + k \sin x \sin a$
$k \cos a = -3, k \sin a = 2$
$k = \sqrt{13}, \tan a = -\frac{2}{3}$
$a = 146.3$
$\cos(x - 146.3) = 0.693$
$x - 146.3 = 46.1, 313.9$
$x = 192.4, 460.2$
$x = 192.4, 100.2$

$k \sin(x + a)$
$k \sin x \cos a + k \cos x \sin a$
$k \cos a = 2, k \sin a = -3$
$k = \sqrt{13}, \tan a = -\frac{3}{2}$
$a = 303.7$
$\sin(x + 303.7) = 0.693$
$x + 303.7 = 43.9, 136.1$
$x = -259.8, -167.6$
$x = 100.2, 192.4$

$k \cos(x + a)$
$k \cos x \cos a - k \sin x \sin a$
$k \cos a = -3, k \sin a = -2$
$k = \sqrt{13}, \tan a = \frac{2}{3}$
$a = 213.7$
$\cos(x + 213.7) = 0.693$
$x + 213.7 = 46.1, 313.9$
$x = -167.6, 100.2$
$x = 192.4, 100.2$

PSfrag replacements
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[SQA] 6. $f(x) = 2\cos x° + 3\sin x°.$

(a) Express $f(x)$ in the form $k\cos(x - \alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. **(4)**

(b) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	Source
(a)	4	3.4			4				3.4.1		1996 Paper 2
(b)	3	3.4			3				3.4.2		Qu.7

(a) $\bullet^1$ $k\cos x\cos\alpha + k\sin x\sin\alpha$

$\bullet^2$ $k\cos\alpha = 2$ **and** $k\sin\alpha = 3$

$\bullet^3$ $k = \sqrt{13}$

$\bullet^4$ $\alpha = 56.3$

(b) $\bullet^5$ $\cos(x - 56.3)° = \frac{0.5}{\sqrt{13}}$

$\bullet^6$ $x - 56.3 = 82.0,\ 278.0$

$\bullet^7$ $x = 138.3,\ 334.3$

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[SQA] 7. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	6	A/B	CN	T14	max value $\sqrt{2}$ when $x = \frac{7\pi}{4}$	2000 P1 Q10

$\bullet^1$ ss: use e.g. $k\cos(x + a)$

$\bullet^2$ ic: expand chosen rule

$\bullet^3$ pd: compare coefficients

$\bullet^4$ pd: process

$\bullet^5$ pd: process

$\bullet^6$ ic: interpret trig expression

$\bullet^1$ e.g. use $k\cos(x + a)$

$\bullet^2$ $k\cos x\cos a - k\sin x\sin a$

$\bullet^3$ $k\cos a = 1$ and $k\sin a = 1$

$\bullet^4$ $k = \sqrt{2}$

$\bullet^5$ $\tan a = 1, a = \frac{\pi}{4}$ (45° *is bad form*)

$\bullet^6$ max. value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$ (*do not accept* 45°)

replacements

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[SQA]   8.

    (a) Show that $2\cos(x° + 30°) - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$.   **3**

    (b) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x° + α°)$ where $k > 0$ and $0 \le α \le 360$ and find the values of k and $α$.   **4**

    (c) Hence, or otherwise, solve the equation $2\cos(x° + 30°) = \sin x° + 1$, $0 \le x \le 360$.   **3**

part	marks	Unit	non-calc C	non-calc A/B	calc C	calc A/B	calc neut C	calc neut A/B	Content Reference : Main Additional	3.4
(a)	3	2.3			3				2.3.2, 1.2.11	Source
(b)	4	3.4			4				3.4.1	**1990 Paper 2**
(c)	3	3.4						3	3.4.2	**Qu. 5**

(a) •^{1} $\cos(x+30)° = \cos x°\cos 30° - \sin x°\sin 30°$

    •^{2} $\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°$

    •^{3} $2 \times \left(\frac{\sqrt{3}}{2}\cos x° - \frac{1}{2}\sin x°\right) - \sin x°$

(b) •^{4} $k\cos x°\cos α° - k\sin x°\sin α°$

    •^{5} $k\sin α° = \sqrt{3}$ and $k\sin α° = 1$

    •^{6} $k = \sqrt{7}$ $\overrightarrow{OG} = 426$

    •^{7} $α = 49.1$

(c) •^{8} $\sqrt{7}\cos(x + 49.1)° = 1$

    •^{9} $x = 18.7°$

    •^{10} $x = 243.1°$

[SQA] 9. The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight.

(a) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)**

(b) (i) Use your result from part (a) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$.

(ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)**

(c) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)**

(d) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	3.4			4				3.4.1		Source
(b)	6	1.2			2	4			1.2.3,	1.2.4	1989 Paper 2
(c)	1	0.1				1			0.1		Qu. 9
(d)	2	0.1				2			0.1		

(a) •¹ $k\cos 30t° \cos\alpha° + k\sin 30t° \sin\alpha°$

•² $k\cos\alpha° = 1$ and $k\sin\alpha° = \sqrt{3}$

•³ $k = 2$

•⁴ $\alpha = 60$

(c) •¹¹ 0800 hours

(d) •¹² 5.6 hours and 10.4 hours

•¹³ e.g. between 5am and 11am

(b) •⁵ maximum at $(2, 2)$

•⁶ minimum at $(8, -2)$

•⁷ endpoints: $(0, 1)$ or $(12, 1)$

•⁸ graph correctly annotated with 3 points

•⁹ sketch with original amplitude increased by factor of 60

•¹⁰ sketch with original graph translated $\begin{pmatrix} 0 \\ 200 \end{pmatrix}$

[SQA] 10. The displacement, d units, of a wave after t seconds, is given by the formula
$d = \cos 20t° + \sqrt{3}\sin 20t°$.

   (a) Express d in the form $k\cos(20t° - \alpha°)$, where $k > 0$ and $0 \leq \alpha \leq 360$.   **4**

   (b) Sketch the graph of d for $0 \leq t \leq 18$.   **4**

   (c) Find, correct to one decimal place, the values of t, $0 \leq t \leq 18$, for which the displacement is 1·5 units.   **3**

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	3.4			4				3.4.1		Source
(b)	4	1.2			2	2			1.2.3		1991 Paper 2
(c)	3	2.3			1	2			2.3.1		Qu. 8

(a) •¹ $k\cos 20t° \cos\alpha° + k\sin 20t° \sin\alpha°$

    •² $k\cos\alpha° = 1$ and $k\sin\alpha° = \sqrt{3}$

    •³ $k = 2$

    •⁴ $\alpha = 60$

(b) •⁵   endpoints: $(0,1)$ or $(18,1)$

    •⁶   zeros: $(7.5,0)$ and $(16.5,0)$

    •⁷   stationary points: $(3,2)$ and $(12,-2)$

    •⁸   correct annotation of graph

(c) •⁹ $2\cos(20t - 60)° = 1.5$

    •¹⁰ $20t - 60 = 41.4 \Rightarrow t = 5.1$

    •¹¹ $20t - 60 = -41.4 \Rightarrow t = 0.9$

[SQA] 11. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$ **4**

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC4
(*a*)	4	C	NC	T13	$\sqrt{2}\sin(x - \frac{\pi}{4})$	2002 P1 Q9
(*b*)	3	C	NC	T15, T14	sketch	

|	
•^{1} ss: know to expand, and expand •^{2} ic: compare coefficients •^{3} pd: write down the value of k •^{4} pd: process a •^{5} ic: sketch a sine curve •^{6} ic: int/com max. and min. values •^{7} pd: process intercepts	•^{1} $k\sin x \cos a - k\cos x \sin a$ *stated explicitly* •^{2} $k\cos a = 1$ **and** $k\sin a = 1$ *stated explicitly* •^{3} $k = \sqrt{2}$ •^{4} $a = \frac{\pi}{4}$ *accept in degrees* •^{5} correct shape of graph (i.e. sin) but not passing through the origin •^{6} graph lies between $\sqrt{2}$ and $-\sqrt{2}$ •^{7} $(\frac{\pi}{4}, 0), (\frac{5\pi}{4}, 0), (0, -1)$ *accept only answers in radians*

[SQA] 12. The function f is defined by $f(x) = 2\cos x° - 3\sin x°$.

(a) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x+\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)**

(b) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \le x < 360$. **(4)**

(c) Write down the minimum value of $(f(x))^2$. **(1)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference : Main Additional	3.4
			C	A/B	C	A/B	C	A/B		Source
(a)	4	3.4			4				3.4.1	1998 Paper 2
(b)	4	3.4			1	3			3.4.3	Qu. 7
(c)	1	0.1				1			0.1	

(a) •¹ $k\cos x\cos\alpha - k\sin x\sin\alpha$ **stated explicitly**

•² $k\sin\alpha = 3$ *and* $k\cos\alpha = 2$ **stated explicitly**

•³ $k = \sqrt{13}$

•⁴ $\alpha = 56.3$

(b) •⁵ $\sqrt{13}\cos(x + 56.3)$

•⁶ Max $= \sqrt{13}$ *and* min $= -\sqrt{13}$

•⁷ $x = 303.7$ and no further answers

•⁸ $x = 123.7$ and no further answers

(c) •⁹ Min Value $= 0$

[SQA] 13. A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.
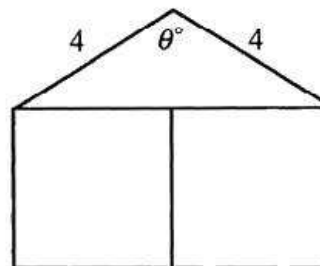
(a) If $\theta°$ is the angle shown in the diagram and A is the area (in square metres) of the gable end, show that $A = 8\left(2 + \sin\theta° - 2\cos\theta°\right)$. **(5)**

(b) Express $8\sin\theta° - 16\cos\theta°$ in the form $k\sin(\theta - \alpha)°$. **(4)**

(c) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**

part	marks	Unit	non-calc		calc		calc neut		Content Reference:		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	5	0.1			1	4			0.1,	2.3.3	Source
(b)	4	3.4			4				3.4.1		1993 Paper 2
(c)	4	3.4			1	3			3.4.2		Qu.9

(a) •¹ area of triangle = $\frac{1}{2} \times 4 \times 4\sin\theta$ or $2 \times \frac{1}{2} \times 4\sin\frac{\theta}{2} \times 4\cos\frac{\theta}{2}$

•² strategy for finding length of side of square or rectangle

•³ for length of side or (length of side)² of square/rectangle

•⁴ area of rectangle

•⁵ simplifying

Note: For •³ various forms of the length are

(b) •⁶ strategy including expansion of $k\sin(\theta - \alpha)$

square: $4\sin\frac{\theta}{2}$, $\frac{2\sin\theta}{\sin(90-\frac{\theta}{2})}$, $\sqrt{16 - 16\cos^2\frac{\theta}{2}}$

•⁷ $k\cos\alpha = 8$ & $k\sin\alpha = 16$

•⁸ $k = 8\sqrt{5}$ or equiv.

rect: $\frac{4\sin\theta}{\sin(90-\frac{\theta}{2})}$, $\sqrt{32 - 32\cos\theta}$

•⁹ $\tan\alpha = 2 \Rightarrow \alpha = 63\cdot4$

(c) •¹⁰ $8(2 + \sin\theta - 2\cos\theta) = 30$

•¹¹ $8\sqrt{5}\sin(\theta - 63\cdot4)° = 14$

•¹² $\sin(\theta - 63\cdot4)° = 0\cdot783$

•¹³ $\theta = 51\cdot5 + 63\cdot4 = 114\cdot9$

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[SQA] 14. (a) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b \sin ax°$, where a and b are constants.
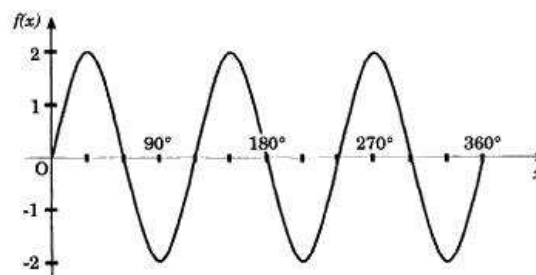Write down the values of a and b.

Diagram 1

(ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d \cos cx°$, where c and d are constants.
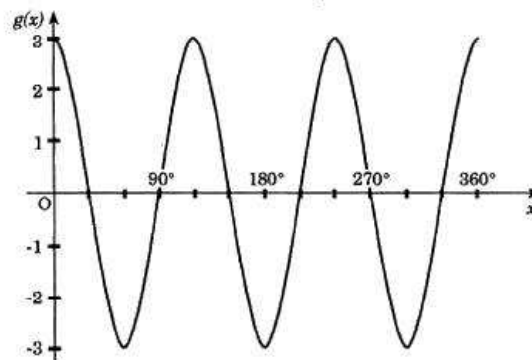Write down the values of c and d.

(4)

Diagram 2

(b) The function h is defined by $h(x) = f(x) + g(x)$.
Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q \sin(px + r)°$ and find the values of p, q and r.

(5)

| part | marks | Unit | non-calc | | calc | | calc neut | | Content Reference : | | 3.4 |
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	2.3			4				2.3.2		Source 1995 Paper 2
(b)	5	3.4			5				3.4.1		Qu.4

(a) •¹ $a = 3$

•² $b = 2$

•³ $c = 3$

•⁴ $d = 3$

(b) •⁵ $p = 3$

•⁶ $q \sin(px + r)°$
$= q \sin px° \cos r° + q \cos px° \sin r°$

•⁷ $q = \sqrt{13}$

•⁸ $q \cos r° = 2, \ q \sin r° = 3$
or $\tan r° = \frac{3}{2}$

•⁹ $r = 56 \cdot 3$

[SQA] 15. An artist has been asked to design a window made from pieces of coloured glass with different shapes. To preserve a balance of colour each shape must have the **same** area. Three of the shapes used are drawn below.
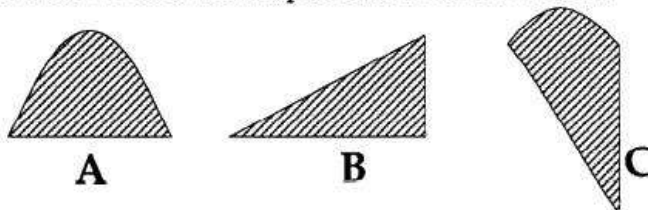
Relative to x,y–axes, the shapes are positioned as shown below.
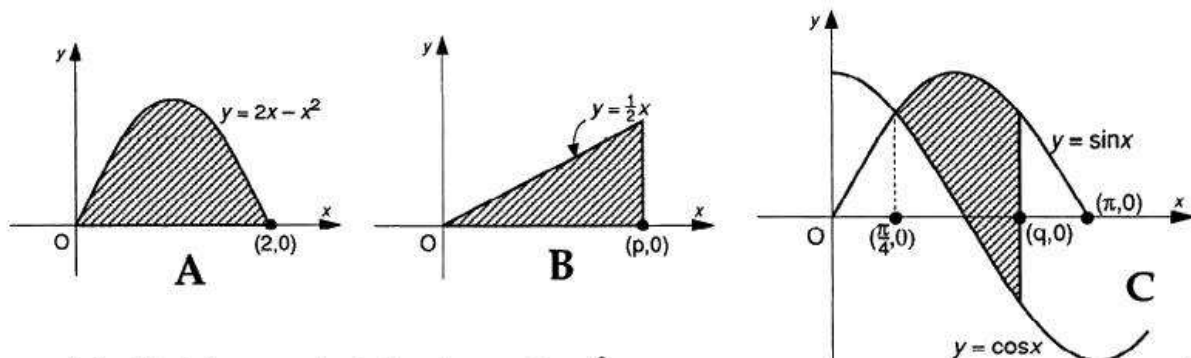
(a) Find the area shaded under $y = 2x - x^2$. (4)

(b) Use the area found in part (a) to find the value of p. (2)

(c) Prove that q satisfies the equation $\cos q + \sin q = 0.081$ and hence find the value of q to 2 significant figures. (10)

part	marks	Unit	non-calc		calc		calc neut		Content Reference :		3.4
			C	A/B	C	A/B	C	A/B	Main	Additional	
(a)	4	2.2			4				2.2.6		Source 1992 Paper 2 Qu.10
(b)	2	0.1			2				0.1		
(c)	10	3.4			2	8			3.4.2,	3.2.1, 2.2.7	

(a) •^{1} strategy: know to integrate

•^{2} $\int_0^2 \left(2x - x^2\right)\, dx$

•^{3} $x^2 - \frac{1}{3}x^3$

•^{4} $1\frac{1}{3}$ $units^2$

(b) •^{5} strategy: use area to find p

•^{6} $p = \frac{4}{\sqrt{3}}$ or equivalent

(c) •^{7} $\int (\sin x - \cos x)\, dx$

•^{8} for the limits $\int_{\frac{\pi}{4}}^{q}$

•^{9} $[-\cos x - \sin x]$

•^{10} $-\cos q - \sin q + \sqrt{2}$

•^{11} $\sqrt{2} - \frac{4}{3} = 0.081$

•^{12} strategy: eg $k\cos(q - \alpha)$

•^{13} $k = \sqrt{2}$

•^{14} $\alpha = \frac{\pi}{4}$

•^{15} $\cos\left(q - \frac{\pi}{4}\right) = \frac{0.081}{\sqrt{2}}$

•^{16} $q = 2.3$

[END OF WRITTEN QUESTIONS]